Math 194
The Invertible Matrix Theorem (Unit 5 Version)

Let $A$ be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given $A$, the statements are either all true or all false.

1. $A$ is an invertible matrix.
2. $A$ is row equivalent to the $n \times n$ identity matrix.
3. $A$ has $n$ pivot positions.
4. $A$ has a pivot in each column.
5. $A$ has a pivot in each row.
6. The equation $Ax = 0$ has only the trivial solution.
7. The equation $Ax = b$ has at least one solution for each $b$ in $\mathbb{R}^n$.
8. The equation $Ax = b$ has exactly one solution for each $b$ in $\mathbb{R}^n$.
9. The columns of $A$ span $\mathbb{R}^n$.
10. The columns of $A$ are linearly independent.
11. The columns of $A$ for a basis for $\mathbb{R}^n$.
12. The nullspace of $A$ equals $\{0\}$.
13. The dimension of the nullspace of $A$ is 0.
14. The column space of $A$ is $\mathbb{R}^n$.
15. The dimension of the column space of $A$ is $n$.
16. The row space of $A$ is $\mathbb{R}^n$.
17. The dimension of the row space of $A$ is $n$.
18. The rank of $A$ is $n$.
19. The linear transformation $x \mapsto Ax$ is onto.
20. The linear transformation $x \mapsto Ax$ is one-to-one.
21. The determinant of $A$ is not zero.
22. The number 0 is not an eigenvalue of $A$. 