1. Suppose \( A \) and \( B \) are \( n \times n \) matrices and that the matrices \( B \) and \( AB \) are invertible. Show that \( A \) is invertible. (Note: You cannot say that \((AB)^{-1} = B^{-1}A^{-1}\), since we don’t know that \( A \) is invertible yet.)

2. Let 
\[
A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix}.
\]

(a) Find elementary matrices \( E, F, \) and \( G \) such that \( GFEA = U \), where \( U \) is an upper triangular matrix (that is, all the entries of \( U \) below the main diagonal are 0).

(b) Find \( L = E^{-1}F^{-1}G^{-1} \). What kind of matrix is \( L \)?

3. Find the inverse of the matrix \( A = \begin{bmatrix} -1 & -3 & -3 \\ 2 & 6 & 1 \\ 3 & 8 & 3 \end{bmatrix} \) by hand (without using Wolfram Alpha or any other similar tool).

4. Let \( A \) be a \( 3 \times 3 \) matrix and suppose that 
\[
2a_1 + a_2 - 4a_3 = 0.
\]

Is \( A \) invertible? Justify your answer.

5. Suppose that \( B \) is a \( 3 \times 4 \) matrix with the property that \( 3b_1 - b_2 + 5b_3 + 7b_4 = 0 \), and suppose that \( A \) is a \( 3 \times 3 \) matrix. Show that the columns of \( AB \) are linearly dependent.

6. Determine whether each of the following sets of vectors is linearly independent. Justify your answers.

(a) \[
\begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 12 \\ 1 \\ -5 \end{bmatrix}
\]

(b) \[
\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 7 \end{bmatrix}
\]

(c) \[
\begin{bmatrix} 5 \\ 10 \\ 0 \\ 15 \end{bmatrix}, \begin{bmatrix} 10 \\ 20 \\ 30 \\ 5 \end{bmatrix}
\]

(d) \[
\begin{bmatrix} a \\ 2a \\ a \end{bmatrix}, \begin{bmatrix} b \\ b \\ b \end{bmatrix}, \begin{bmatrix} 2a + 3b \\ 4a + b \\ 2a + b \end{bmatrix}, \text{ where } a \text{ and } b \text{ are particular constants}
\]

(e) \[
\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -24 \\ 11 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}
\]
7. Suppose that \( \mathbf{v}_1, \mathbf{v}_2, \) and \( \mathbf{v}_3 \) form a linearly independent set of vectors in \( \mathbb{R}^3 \). Suppose also that the vector \( \mathbf{u} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 \) and that \( \mathbf{u} = b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + b_3\mathbf{v}_3 \). True or False: It must be the case that \( a_1 = b_1, \ a_2 = b_2, \) and \( a_3 = b_3 \). Justify your answer.

8. Let \( A \) be a \( 4 \times 4 \) matrix with reduced row echelon form given by \( U = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \). If the first column of \( A \) is \( \begin{bmatrix} -3 \\ 5 \\ 2 \\ 1 \end{bmatrix} \) and the second column of \( A \) is \( \begin{bmatrix} 4 \\ -3 \\ -7 \\ -1 \end{bmatrix} \), determine the third and fourth columns.