1. Solve each of the following systems of linear equations by hand using Gaussian elimination.

(a)
\[
\begin{align*}
x - y + 2z &= 5 \\
2x - 2y + 4z &= 10 \\
3x - 3y + 6z &= 15
\end{align*}
\]

(b)
\[
\begin{align*}
x - 2y + z &= 0 \\
2y - 8z &= 8 \\
-4x + 5y + 9z &= -9
\end{align*}
\]

2. Given a system of the form
\[
\begin{align*}
-m_1 x + y &= b_1 \\
-m_2 x + y &= b_2
\end{align*}
\]
where \(m_1, m_2, b_1,\) and \(b_2\) are constants,

(a) Show algebraically that the system will have a unique solution if \(m_1 \neq m_2\).

(b) Show that if \(m_1 = m_2\), then the system will have a solution only if \(b_1 = b_2\).

(c) Give a geometric interpretation of parts (a) and (b).

3. Construct three different augmented matrices for linear systems whose solution set is \(x_1 = -2, x_2 = 1,\) and \(x_3 = 0\).

4. Suppose you have a simple board game called Triad consisting of three spaces (A, B, and C) arranged in a triangle. Each round you move your playing piece according to the following rule: There’s a 50% chance you’ll stay where you are, a 30% chance you’ll move forward one space, and a 20% chance you’ll move backward one space. (Assume that forward = clockwise around the triangle and backward = counterclockwise.)

(a) Considering this game as a Markov chain, construct the transition matrix \(P\).

(b) What does the (2,3) entry in \(P\) represent?