More Practice Problems

1. Find all integer solutions to the system of modular arithmetic equations:
   y ≡ 2x + 3 (mod 10)
   y ≡ 4x + 1 (mod 10)

2. Suppose a certain cipher machine has a set of five scramblers, each of which can be set in one of 26 orientations (A-Z), like the Enigma machine.   Unlike the Enigma machine, these scramblers are bolted into the machine and can't be rearranged. However, each scrambler can be set to "active," in which case it affects encryption, or "inactive," in which case it doesn't. Thus, a key for this cipher machine consists of some subset of the five scramblers that are active, along with an orientation (A-Z) for each active scrambler.
   a. How many possible keys are there in which three of the scramblers are active?
   b. Suppose one of the scramblers is broken and stuck on the J orientation. Then how many possible keys are there in which three of the scramblers are active? (Note that the broken scrambler might be active, but it might not.)

3. Suppose the 8 letters V, I, G, E, N, E, R, and E are each written on a tile and placed in a bag. (You can imagine the game Scrabble, if that helps.) If you reach into the bag and draw <u>five</u> tiles at random (without replacement), what is the probability that…
   a. You draw no Es?
   b. You draw exactly 1 E?
   c. You draw at least one E?

4. Suppose the 8 letters V, I, G, E, N, E, R, and E are each written on a tile and placed in a bag. (You can imagine the game Scrabble, if that helps.) If you reach into the bag and draw <u>six</u> tiles at random (without replacement), what is the probability that…
   a. You draw all three Es?
   b. You draw exactly 2 Es?
   c. You draw no Es?

5. Suppose x is a two-digit positive integer when represented in decimal form. How many digits could x have…
   a. When represented in binary form?
   b. When represented in base-3?
   c. When represented in hexadecimal (base-16)?

6. A certain website requires that users create passwords that have exactly 8 characters. Each character can be a lowercase letter (a-z) or a digit (0-9). A password cannot consist entirely of letters, nor can it consist entirely of digits. How many possible passwords are there?

7. A military radio operator is intercepting communications from opposing forces. There's a 40% chance that a given intercept is encrypted. (Encrypted communications are sent by the radio operator to his unit's codebreaking division; unencrypted ones aren't interesting and are discarded.) If the radio operator gets to take a break after he intercepts four encrypted communications, what is the probability that he will get to take a break immediately after the eighth intercept he makes during his shift?