

Math 115F Math Exam Study Guide

This week's math exam will cover the mathematical concepts and techniques we've explored this semester. The exam will not involve any codebreaking, although some questions on the exam may draw on cryptography for context. You are encouraged to bring a calculator (scientific or graphing) to the test, but you will not be allowed to use a laptop during the test.

You should be able to do each of the tasks listed below and understand the concepts associated with each task.

- Modular Arithmetic
 - Determine if two integers are congruent modulo a given integer m .
 - Generate a set of integers all congruent to a given integer x modulo a given integer m .
 - Simplify or solve a modular arithmetic equation.
 - Solve a system of modular arithmetic equations.
 - Calculate $x \text{ MOD } m$, given integers x and m .
- Prime Numbers
 - Determine if a given number is prime.
 - Find the prime factorization of a given composite number.
 - Determine if two given numbers are relatively prime.
 - Find numbers that are relatively prime to a given number.
- Common Divisors
 - Find the common divisors of a set of integers (as in the Kasiski Test).
 - Determine the greatest common divisor of two integers using the Euclidean Algorithm.
 - Express the greatest common divisor of two integers as an integer multiple of one plus an integer multiple of the other.
- Combinatorics
 - Calculate the number of permutations of r objects from a set of n objects.
 - Determine the number of unique permutations of a sequence of letters, with or without repeated letters.
 - Calculate the number of combinations of r objects from a set of n objects.
 - Calculate the number of possibilities for a given scenario using a mix of permutations and/or combinations.
- Probability
 - Compute probabilities for experiments with equally likely outcomes.
 - Compute probabilities using the basic rules of probability. (See next page.)
- Binary Numbers
 - Convert a number from decimal to binary representation.
 - Convert a number from binary to decimal representation.
 - Add or subtract binary numbers.
 - More generally, convert from decimal representation to representation in a different base, and vice versa.

Basic Rules of Probability

- SUM RULE: If events A and B are mutually exclusive, then the probability of A or B occurring equals $P(A) + P(B)$.
- PRODUCT RULE: If events A and B are independent, then the probability of A and B occurring is $P(A) \cdot P(B)$.
- COMPLEMENT RULE: The probability of event A not occurring is $1 - P(A)$.

Practice Problems

1. Determine five *integer* solutions to each of the following equations.
 - a. $x - 4 \equiv 5 \pmod{26}$
 - b. $x + 23 \equiv 1 \pmod{4}$
 - c. $5x \equiv 1 \pmod{8}$
 - d. $3x + 1 \equiv 4 \pmod{5}$
2. Calculate the following.
 - a. $14 \text{ MOD } 3$
 - b. $130 \text{ MOD } 26$
 - c. $-1 \text{ MOD } 5$
 - d. $-258 \text{ MOD } 16$
3. Determine the prime factorization of the following numbers.
 - a. 961
 - b. 2310
 - c. 6517
4. Find three numbers that are relatively prime to each of the following numbers.
 - a. 75
 - b. 120
 - c. 310
 - d. 512
5. Find all common divisors for each of the following sets of numbers.
 - a. 42, 70, 126, and 154
 - b. 50, 125, 275, and 300
 - c. 52, 130, 182, and 468
6. Use the Euclidean algorithm to find the greatest common divisor of each pair of integers. That is, find $\text{gcd}(a, b)$.
 - a. $a = 667$ and $b = 437$
 - b. $a = 3001$ and $b = 541$

- c. $a = 77897$ and $b = 3721$
7. For each of the following pairs a and b , find integers s and t such that $as + bt = \gcd(a, b)$.
- $a = 667$ and $b = 437$
 - $a = 3001$ and $b = 541$
 - $a = 77897$ and $b = 3721$
8. If the letters B, C, D, F, G, H, and J are written on seven index cards...
- How many three-letter "words" can be formed?
 - How many five-letter "words"?
 - In how many ways can three of these cards be selected?
 - In how many ways can five of them be selected?
9. Given a standard 52-card deck (that is, cards ranked Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King, in each of four different suits—hearts, diamonds, clubs, and spades), determine the number of each type of hand listed below that are possible on a 5-card draw.
- Full House – 3 cards of one rank, 2 cards of another rank
 - Flush – 5 cards of the same suit
 - Straight – 5 cards of consecutive ranks (ex.: 8, 9, 10, Jack, Queen)
 - Three-of-a-Kind – 3 cards of one rank, 2 cards of other ranks
10. Tennessee auto license plates have three letters followed by three digits.
- How many different Tennessee plates are possible?
 - If two Tennessee plates are selected at random, what is the probability that they will have the same three digits?
 - How many different Tennessee plates include the letters Q, X, and Z?
 - What is the probability that a randomly selected Tennessee plate will include the letter D?
11. Using binary representations, calculate $a + b$ and $a - b$.
- $a = 11\ 011$, $b = 10\ 101$
 - $a = 1\ 111\ 11$, $b = 101$
12. Decimal representations use base 10. Binary representations use base 2. Find the decimal representation of each of the following numbers represented in base 3.
- 201
 - 111
 - 21 001