Example:

Find all integer solutions to the system

$$\begin{cases} y \equiv 2x+3 \pmod{10} \\ y \equiv 4x+1 \pmod{10} \end{cases}$$

Congruence is transitive, so

$$2x+3 \equiv 4x+1 \pmod{10}$$

$$2 \equiv 2x \pmod{10}$$

Thus $2x = \ldots, -8, 2, 12, 22, 32, \ldots$

and $x = \ldots, -4, 1, 6, 11, 16, \ldots$

x can be any of these numbers.

Note: if you "simplify"

$$2 \equiv 2x \pmod{10}$$

to

$$1 \equiv x \pmod{10}$$

you'll get $x = \ldots, 1, 11, 21, \ldots$ and miss "half" the solutions. Dividing both sides of a congruence equation by a number that has a factor in common with the modulus doesn't work.

Suppose $x = 1$. Then

$$y \equiv 2(1)+3 \pmod{10}$$

$$y \equiv 5 \pmod{10}$$

$$y = \ldots, -5, 5, 15, 25, \ldots$$

Suppose $x = 6$. Then

$$y \equiv 2(6)+3 \pmod{10}$$

$$y \equiv 15 \pmod{10}$$

$$y = \ldots, -5, 5, 15, 25, \ldots$$

In general, if $2x \equiv 2 \pmod{10}$, then $y \equiv 2+3 \pmod{10}$

Thus, we have solutions:

(x, y) where $x = \ldots, -4, 1, 6, 11, 16, \ldots$
and $y = \ldots, -5, 5, 15, 25, \ldots$

sample solutions: $(1,5)$ $(1,15)$ $(-4,15)$ $(6,25)$ $(116,75)$