Math 216 Spring 2012
Problem Set 6 Answer Key

1. (a) Assume $p = 0.5$, since this will give us the maximum SE possible. We have

$$\begin{aligned} 0.05 &= z^* \sqrt{\frac{p(1-p)}{n}} \\ 0.05 &= 2.33 \sqrt{\frac{0.25}{n}} \end{aligned}$$

Solve for n: $n = 542.89$, so we must take a sample of at least **543** observations.

(b) $n = 200$, $\hat{p} = 30/200 = 0.15$

$$\begin{aligned} \text{Confidence Interval} &= \hat{p} \pm z^* \text{SE}_{\hat{p}} \\ &= \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= 0.15 \pm 2.33 \sqrt{\frac{.15(.85)}{200}} \\ &= 0.15 \pm 0.0588 \\ &= (\mathbf{0.0912}, \mathbf{0.2088}) \end{aligned}$$

2. $n = 52, \hat{p} = 0.9$
Width of new confidence interval is

$$2(z^* \text{SE}_{\hat{p}}) = 2(1.96) \sqrt{\frac{.9(.1)}{52}} = 2(0.0815) = \mathbf{0.163}$$

Width of the original confidence interval was $0.557 - 0.299 = \mathbf{0.258}$, so the new confidence interval is **narrower**. There is (apparently) less variance in the population if the sample proportion is 0.9, so our estimate is more precise.

3. We are given: $n = 986$, and margin of error $= 0.03$. We want to find the confidence level.

$$\begin{aligned} \text{ME} &= z^* \frac{0.5}{\sqrt{n}} \\ 0.3 &= z^* \frac{0.5}{\sqrt{986}} \end{aligned}$$

Solve: $z^* \approx 1.88$
$P(-1.88 \leq z \leq 1.88) = \mathbf{94}\%$ confidence level.

4. H_0: $p = 0.3$
   H_A: $p < 0.3$

$$\begin{aligned} p\text{-value} &= P(\hat{p} < 0.267 \mid p = 0.3) \\ &= \left(z < \frac{0.267 - 0.3}{\sqrt{0.3(0.7)/300}} \right) \\ &= P(z < -1.247) = \mathbf{0.1056} \end{aligned}$$

   If his "true" batting average is 0.300, he would still have a proportion of hits to at-bats this lousy 10.56% of the time. We don't have strong evidence that his true batting average is less than 0.3.

5. p_1 = proportion of small cars that would be totaled
   p_2 = proportion of large cars that would be totaled
   $\hat{p_1} = \frac{16}{24} \approx .667$
   $\hat{p_2} = \frac{10}{30} \approx .333$
   $\hat{p_1} - \hat{p_2} \approx 0.333$
   Hypothesis test
   H_0: $p_1 - p_2 = 0$
   H_A: $p_1 - p_2 \neq 0$

$$\begin{aligned} p\text{-value} &= P(\hat{p_1} - \hat{p_2} \geq 0.333 \text{ or } \hat{p_1} - \hat{p_2} \leq -0.333 \mid p_1 = p_2) \\ &= 2P\left(z \leq \frac{-.333}{\text{SE}} \right) \end{aligned}$$

   SE calculation:

$$\hat{p} = \frac{16 + 10}{54} \approx 0.481$$

$$\text{SE} = \sqrt{\frac{0.481(.519)}{24} + \frac{.481(.519)}{30}} = 0.1368$$

   So we have

$$\begin{aligned} p\text{-value} &= 2P\left(z \leq \frac{-.333}{.1368} \right) \\ &= 2P\left(z \leq -2.43 \right) \\ &= 2(.0075) = \mathbf{.015} \end{aligned}$$

   We can reject H_0 and conclude that $p_1 \neq p_2$.

6. (a) H_0: $\mu_2 - \mu_1 = 0$
   H_A: $\mu_2 - \mu_1 > 0$

$$\begin{aligned} p\text{-value} &= P(\bar{x_2} - \bar{x_1} > 321 - 290 \mid \mu_1 = \mu_2) \\ &= P\left(z > \frac{31 - 0}{\text{SE}_{\bar{x_2} - \bar{x_1}}}\right) \\ &= P\left(z > \frac{31 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}\right) \\ &= P\left(z > \frac{31}{1.772}\right) \\ &= P(z > 17.49) \approx \mathbf{0} \end{aligned}$$

   We can reject H_0 and conclude that, yes, supplier 2 provides gears with higher mean impact strength.

   (b) H_0: $\mu_2 - \mu_1 = 25$
   H_A: $\mu_2 - \mu_1 > 25$

$$\begin{aligned} p\text{-value} &= P(\bar{x_2} - \bar{x_1} > 31 \mid \mu_2 - \mu_1 = 25) \\ &= P\left(z > \frac{31 - 25}{1.772}\right) \\ &= P(z > 3.38) = \mathbf{0.0004} \end{aligned}$$

   We can reject H_0 and conclude that, yes, the mean impact strength of gears from supplier 2 is at least 25 foot-pounds higher than that of gears from supplier 1.

   (c) Confidence Interval: $\bar{x_2} - \bar{x_1} \pm z^* \text{SE}_{\bar{x_2} - \bar{x_1}}$

$$\begin{aligned} &95\% \text{ confidence: } 31 \pm 1.96(1.772) = 31 \pm 3.47 = (\mathbf{27.53}, \mathbf{34.47}) \\ &99\% \text{ confidence: } 31 \pm 2.57(1.772) = 31 \pm 4.55 = (\mathbf{26.45}, \mathbf{35.55}) \end{aligned}$$

7. Let
   μ_1 = average audience score
   μ_2 = average critics score
   $\mu_{\text{diff}} = \mu_1 - \mu_2$

$$\text{From R-studio or Excel: } \begin{array}{lll} \bar{x}_1 = 53.918 & s_1 = 26.99 & \\ \bar{x}_2 = 62.192 & s_2 = 17.05 & \\ \bar{x}_{\text{diff}} = 8.274 & s_{\text{diff}} = 16.257 & n_{\text{diff}} = 146 \end{array}$$

   Hypothesis test :
   $H_0 : \mu_{\text{diff}} = 0$

$H_A : \mu_{\text{diff}} > 0$

$$
\begin{aligned}
p\text{-value} &= P(\bar{x}_{\text{diff}} > 8.274 \mid \mu_{\text{diff}} = 0) \\
&= P\left(z > \frac{8.274 - 0}{16.257/\sqrt{146}}\right) \\
&= P(z > 6.15) \approx \mathbf{0}
\end{aligned}
$$

Therefore we can reject H_0 and conclude that, yes, the average audience score is greater than the average critics score.