Name: Answer Key

Math 216 Midterm 2

March 28, 2012

Problem Number	Possible Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	16	
8	16	
9	16	
10	16	
Total	100	

Directions—Please Read Carefully!

- Read each problem carefully and make sure to answer the specific questions asked. Some questions ask you to justify or explain your answers. You must do so to receive full credit on these questions.
- You may use a graphing or scientific calculator on this exam, as well as one 3" x 5" index card (front and back) with notes or formulas. No other aids are allowed.

Questions 1 through 6 are multiple-choice questions. Please write your answer choice for each question in the blank that precedes the question.

1. __A__ In a study of the lifetime of electronic components, a random sample of 400 components are tested until they fail to function. Based on the sample mean and sample standard deviation, a 95% confidence interval for the population mean lifetime was found to be 370 ± 63.7 hours. Which of the following statements is **true**?

   A. If the process of computing a 95% confidence interval from 400 sample measures is repeated 100 times, the expected number of such intervals that include the population mean lifetime would be 95.

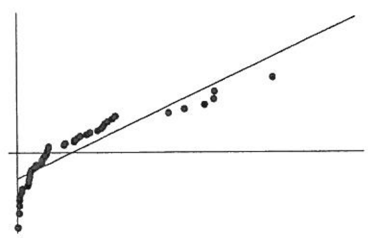
   B. There is only a 5% chance that the sample mean for another sample of 400 measurements will fall outside the interval 370 ± 63.7.

   C. Approximately 95% of all future measurements of the lifetimes of these components will be in the interval 370 ± 63.7 hours.

   D. Approximately 95% of the 400 sample measurements were in the interval 370 ± 63.7 hours.

That's the definition of a confidence interval. Also, options B, C, and D are demonstrably incorrect.

2. __C__ A stress analysis was conducted on a random sample of size 50 of epoxy-bonded joints made from a certain species of wood. The shear stress (in pounds per square inch) of each joint was determined, and this data set was used to construct the following normal probability plot.



Which of the following random variables is approximately normally distributed?

   A. The shear stress of a randomly selected joint of this type

   B. The mean shear stress of a random sample of 15 joints of this type

   C. The mean shear stress of a random sample of 60 joints of this type

   D. Both B and C

This distribution (call it X) is highly non-normal. The sample mean ($\bar{X}$) is normal by the Central Limit Theorem only if X is normal (which it's not here) or the sample size is large.

3. C Suppose you construct a 95% confidence interval from a random sample of size 10, but you find that the interval is too wide to be useful. Which of the following steps **must** result in a **narrower** confidence interval?

A. Take another, larger random sample and use it to construct a 95% confidence interval.

B. Take another, smaller random sample and use it to construct a 95% confidence interval.

C. Use the sample mean and sample standard deviation from your current sample to construct a 90% confidence interval.

D. Use the sample mean and sample standard deviation from your current sample to construct a 99% confidence interval.

New samples could yield wider CIs, so A and B are out. Since the width of a CI equals

$$2 z^* s/\sqrt{n}$$

we'll need z^* to be smaller, which occurs when the confidence level decreases.

Or: less confident ⇒ more precise

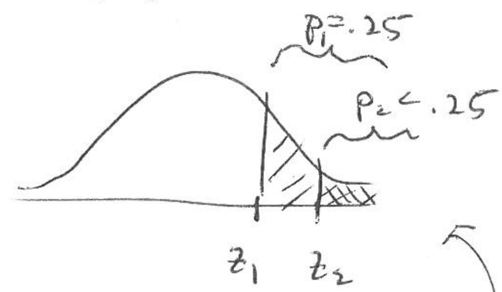
4. A Suppose that a researcher suspects that the average IQ of a Vanderbilt student is greater than 100, so she takes a random sample of Vanderbilt students and determines the IQ of each student in the sample. She uses this sample to conduct the hypothesis test $H_0 : \mu = 100$ versus $H_A : \mu > 100$ and determines that the p-value for this test is $p = 0.25$. She concludes that this is not convincing evidence that the average IQ of a Vanderbilt student is greater than 100, so she takes a second random of the same size as the first one. The mean for the second sample is the same as the mean for the first sample, but the standard deviation for the second sample is *smaller than* that of the first sample. Which of the following statements is **true**?

A. The p-value for the second sample will be **smaller than** 0.25.

B. The p-value for the second sample will be **larger than** 0.25.

C. The p-value for the second sample will be **equal to** 0.25.

D. There is no way to determine the relationship between the p-values for the two samples given the available information.

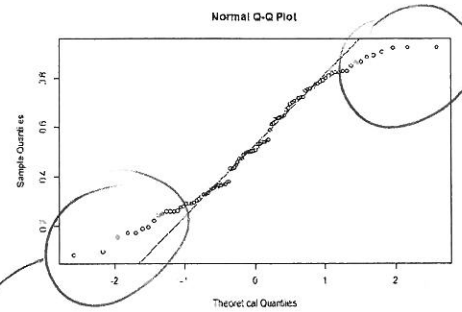


$$p = P(\bar{X} > \bar{x} \mid \mu = 100)$$

$$= P\left(Z > \underbrace{\frac{\bar{x} - 100}{s/\sqrt{n}}}\right)$$

smaller s ⇒ bigger z-score ⇒ smaller p-value

5. ___D___ Which of the following statements is true about the distribution that generated the following normal probability plot?



→ These observations are smaller than expected, which means the tail drops off faster.
⇒ short right tail

A. Its left tail is longer than that of a normal distribution, while its right tail is shorter.
B. Its left tail is shorter than that of a normal distribution, while its right tail is longer.
C. Both of its tails are longer than the corresponding tails of a normal distribution.
D. Both of its tails are shorter than the corresponding tails of a normal distribution.

These observations aren't as small as expected, which means the tail isn't as long.
⇒ short left tail

6. ___C___ Suppose a class consists of 18 civil engineering majors and 12 environmental engineering majors. Ten of these students are selected at random *without replacement*. Let X be the number of students among the ten who are environmental engineering majors. Then X is not a binomial random variable. Which of the following conditions, all of which are necessary for binomial random variables, is not satisfied by X?

A. The experiment consists of a sequence of n smaller experiments (trials).
B. Each trial can result in one of two outcomes–success or failure.
C. The trials are independent.
D. The probability of success, p, is constant from trial to trial.

The probability that any one of the ten students is an EE major is $\frac{12}{30}$ or .4. However, knowing that one of the ten is an EE major means that the conditional prob. of another one being an EE major is less than .4. Thus the trials are not independent. (If the population were much larger, we could ignore this.)

The following questions are free-response questions. For these questions, the more of your work you write down, the more easily I can grant you partial credit if your answer is incorrect.

7. Suppose the amount of sugar contained in 1-kg packets is normally distributed with a mean of $\mu = 1.03$ kg and a standard deviation of $\sigma = 0.014$ kg.

   (a) What proportion of sugar packets are underweight? (That is, what proportion weigh less than 1 kg?)

$$P(X < 1) = P\left(Z < \frac{1-1.03}{.014}\right) = P(Z < -2.14)$$

$$= \boxed{.0162}$$

   (b) What proportion of sugar packets weigh within 0.05 kg of the advertised weight?

$$P(.95 < X < 1.05)$$

$$= P\left(\frac{.95-1.03}{.014} < Z < \frac{1.05-1.03}{.014}\right)$$

$$= P(-5.71 < Z < 1.43)$$

$$= P(Z < 1.43) - \underbrace{P(-5.71)}_{\approx 0} \approx \boxed{.9236}$$

8. In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with an average of 12 log-ons per hour.

(a) What is the probability that there will be at most two log-ons in a half hour?

$\lambda = E(X) = 6$ in a half hour $\Rightarrow P(X=k) = \dfrac{6^k e^{-6}}{k!}$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \dfrac{6^0 e^{-6}}{0!} + \dfrac{6^1 e^{-6}}{1!} + \dfrac{6^2 e^{-6}}{2!} = \boxed{.062}$

(b) Let t be the smallest number for which the probability of at least one log-on occuring in a t-minute span is greater than 0.50. What is t?

$t = ?$

$P(X \geq 1) = .5$

$1 - P(X=0) = .5$

$P(X=0) = .5$

$\dfrac{\lambda^0 e^{-\lambda}}{0!} = .5$

$e^{-\lambda} = .5$

$\lambda = -\ln(.5)$

$\cong .693$

$\dfrac{\lambda}{t} = \dfrac{12}{60}$

$\Rightarrow \dfrac{.693}{t} = \dfrac{12}{60}$

$\Rightarrow \boxed{t = 3.47 \text{ min}}$

9. A multiple-choice test contains 8 questions, each with four answers. Assume that a student just guesses on each question so that the student has a 25% chance of answering any question correctly. The student must answer at least three of the 8 questions correctly to pass the test.

(a) What is the probability that the student passes the test?

X = # questions correct

X is Binomial with $n=8$, $p=.25$

$$P(X \geq 3) = 1 - P(X \leq 2)$$

$$= 1 - \left(P(X=0) + P(X=1) + P(X=2)\right)$$

$$= 1 - \left(\binom{8}{0}.25^0 .75^8 + \binom{8}{1}.25^1 .75^7 + \binom{8}{2}.25^2 .75^6\right)$$

$$= \boxed{.3215}$$

(b) Suppose that the student takes five such tests. What is the probability that the student passes at least two of the tests?

Y = # tests passed

Y is Binomial with $n=5$, $p=.3215$

$$P(Y \geq 2) = 1 - P(Y \leq 1)$$

$$= 1 - \left(P(Y=0) + P(Y=1)\right)$$

$$= 1 - \left(\binom{5}{0}.3215^0 .6785^5 + \binom{5}{1}.3215^1 .6785^4\right)$$

$$= \boxed{.513}$$

10. At a particular offshore oil platform, the risk management department determines that workers on the platform need to be able to escape the platform in the event of an emergency in no more that 360 seconds (or six minutes). A sample of 52 platform workers took part in a simulated escape exercise, resulting in an average time to complete the escape of 352 seconds with a standard deviation of 24 seconds. The platform foreman is worried that these promising results were due to random chance, and that the true average escape time for his workers is greater than 360 seconds.

(a) What are appropriate null and alternate hypotheses to test the claim that the true average escape time is more than 360 seconds?

$H_0: \mu = 360$

$H_A: \mu < 360$ → How likely are we to get results this good or better if μ is really 360?

(b) What are the Type 1 and Type 2 errors for this hypothesis test?

Type 1: Assuming the workers are safe when they are not really.

Type 2: Assuming the workers are at risk when they're really safe.

(c) From the perspective of the platform workers, which of these errors is more serious?

Type 1

(d) Compute the p-value for this test.

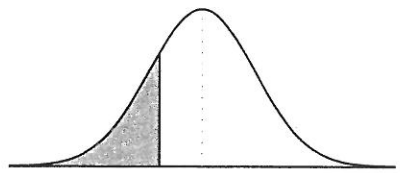
$$p = P(\bar{X} < 352 \mid \mu = 360) = P\left(Z < \frac{352-360}{24/\sqrt{52}}\right)$$

$$= P(Z < -2.40) \approx \boxed{.0082}$$

(e) What should the platform foreman conclude from this simulated escape exercise about the true average escape time for his workers?

Low p-value
⇒ reject H_0
⇒ Conclude the workers are fast enough.

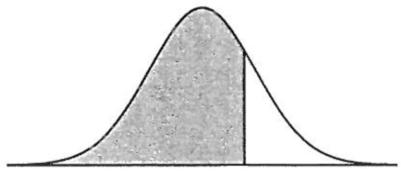
If the workers aren't fast enough, there's only a .82% chance they would do this well in a test. Since they did this well, it's probably because they're fast enough.



negative Z

Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

*For $Z \leq -3.50$, the probability is less than or equal to 0.0002.



positive Z

	Second decimal place of Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

*For $Z \geq 3.50$, the probability is greater than or equal to 0.9998.