Math 216 Spring 2012
Problem Set 4 Answer Key

1. $n = 6$, $p = 0.07$ Define X = number of planes experiencing exceedence in a day.

   (a)
   $$P(X = 1) = \binom{6}{1}(.07)^1(.93)^5 \approx \mathbf{0.292}$$

   (b)
   $$P(X = 0) = 0.93^6 \approx \mathbf{0.647}$$

   (c)
   $$E(X) = np = \mathbf{0.42}$$
   $$\sigma^2(X) = np(1-p) = \mathbf{0.3906}$$
   $$\sigma(X) = \sqrt{0.391} = \mathbf{0.625}$$

2. $n = 20$, $p = 0.04$, X = number of parts in sample of 20 that require rework.

   (a)
   $$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 1 - (P(X = 0) + P(X = 1)) \\ &= 1 - \left[\binom{20}{0}(0.96)^{20} - \binom{20}{1}(0.04)^1(.96)^{19}\right] \\ &= \mathbf{0.19} \end{aligned}$$

   (b) Probability that X exceeds 1 in at least one of the next five hour samples:
   = 1−Probability that X exceeds 1 in none of the next five hour samples
   $= 1 - (1 - P(X > 1))^5$
   $= \mathbf{0.651}$

3. $n = 125, p = 0.1$ Define X to be the number of passengers who do not show up.

   (a)
   $$\begin{aligned} P(X \geq 5) &= 1 - P(X < 5) \\ &= 1 - \sum_{k=0}^{4} \binom{125}{k}(0.1)^k(0.9)^{125-k} \\ &\approx \mathbf{0.996} \end{aligned}$$

(b)

$$\begin{aligned} P(X > 5) &= 1 - P(X \leq 5) \\ &= 1 - \sum_{k=0}^{5} \binom{125}{k} (0.1)^k (0.9)^{125-k} \approx \mathbf{0.989} \end{aligned}$$

4. X: number of flaws in 1000m wire, $X \sim$ Poisson, $\lambda = 2.6$

   (a)

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= \frac{2.6^0 e^{-2.6}}{0!} + \frac{2.6^1 e^{-2.6}}{1!} \approx \mathbf{0.267} \end{aligned}$$

   (b)

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - \sum_{k=0}^{2} \frac{2.6^k e^{-2.6}}{k!} \approx \mathbf{0.482} \end{aligned}$$

   (c) Define X : number of flaws in 500 m of wire, now $\lambda = 1.3$

$$P(X = 0) = \frac{1.3^0 e^{-1.3}}{0!} \approx \mathbf{0.273}$$

5. X : number of accidents in a month. $\lambda = 2.5$.

   (a)

$$P(X = 0) = \frac{2.5^0 e^{-2.5}}{0!} \approx \mathbf{0.0821}$$

   (b)

$$P(X > 1) = 1 - \frac{2.5^0 e^{-2.5}}{0!} + \frac{2.5^1 e^{-2.5}}{1!} \approx \mathbf{0.713}$$

6. X : breaking strength of yarn. $X \sim$Normal, $\sigma = 2$psi, $n = 9$, $\bar{x} = 98$

$$0.95 = P\left(-1.96 \leq \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \leq 1.96\right)$$

   So our 95% confidence interval is

$$\begin{aligned} &= \left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \ , \ \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) \\ &= \left(98 - 1.96\frac{2}{3} \ , \ 98 + 1.96\frac{2}{3}\right) \\ &= (\mathbf{96.693}, \mathbf{99.307}) \end{aligned}$$

7. $n = 120$, $\bar{x} = 178$, $s = 14$.
CLT says since n is large, $\bar{X}$ is approximately normal. Moreover, since $n > 50$, we can say s is a good approximation for σ.

   (a) 99% confidence interval for true mean capacity of batteries:

$$\begin{aligned} &\left(\bar{x} - 2.57\frac{\sigma}{\sqrt{n}} \ , \ \bar{x} + 2.57\frac{\sigma}{\sqrt{n}}\right) \\ =&\left(178 - 2.57\frac{14}{\sqrt{120}} \ , \ 178 + 2.57\frac{14}{\sqrt{120}}\right) \\ =&(\mathbf{174.715} \ , \ \mathbf{181.285}) \end{aligned}$$

   (b) Find n such that

$$2.57\frac{14}{\sqrt{n}} = 2 \implies n = \mathbf{324}$$

8. 95% confidence interval for $\mu_F = (0.213, 0.241)$

$$(0.213, 0.241) = \left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \ , \ \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

Therefore,

$$\begin{aligned} \bar{x} &= 0.227 \\ \frac{\sigma}{\sqrt{n}} &= 0.00741 \end{aligned}$$

So the 90% confidence interval is

$$(0.227 - 1.645(.00714) \ , \ 0.227 + 1.645(.00714)) = (\mathbf{0.2153} \ , \ \mathbf{0.2387})$$

9. X: time needed to reach 100^oF, $\bar{x} = 15.8$, $\sigma = 2.2$

   (a) $H_0 : \mu = 15$
   $H_A : \mu > 15$

   (b)

$$\begin{aligned} p = P(\bar{X} \geq 15.8) = P\left(Z \geq \frac{15.8 - 15}{2.2/\sqrt{30}}\right) \\ = 1 - P(Z \leq 1.99) = 1 - 0.9767 \\ = \mathbf{0.0233} \end{aligned}$$

   (c) Answers may vary. Credit given for correct interpretation of p-value.

10. Answers vary.

$H_A : \mu > 5$ — We reject the null hypothesis only if there is strong evidence that exposure is greater than 5 rem per year. This stance assumes the workplace is safe, so it tends to favor the radiation plant employing the worker. (We might choose this option if we're trying to make life easier for the radiation company)

$H_A : \mu < 5$ — We reject the null hypothesis only if there is strong evidence that exposure is less than 5 rem per year. In other words, we assume the workplace is dangerous unless there is strong evidence that it is safe. This position favors the health of the worker and places the burden of proof on the employer. (This position might be the more ethical one)

11. $n = 1000$, $\bar{x} = 50.1$, $s = 1$

    (a) $H_0 : \mu = 50$
    $H_A : \mu > 50$

    (b)

$$\begin{aligned} p = P(\bar{X} > 50) &= P\left(Z \geq \frac{50.1 - 50}{1/\sqrt{1000}}\right) \\ &= 1 - P(Z \leq 3.16) = 1 - 0.9992 \\ &= \mathbf{0.0008} \end{aligned}$$

    (c) No, although we are reasonably certain that the new method produces stronger fibers ($p < 0.01$), the improvement is very small, which is probably not worth the expense of retooling.