Math 216
Problem Set 4

1. Airplanes approaching the runway for landing are required to stay within the localizer (a certain distance left and right of the runway). When an airplane deviates from the localizer, it is sometimes referred to as an exceedence. Consider one airline at a small airport with six daily arrivals and an exceedence rate of 7%.

   (a) Find the probability that on a particular day exactly one plane experiences an exceedence.

   (b) Find the probability that on a particular day no planes experience an exceedence.

   (c) Find the expected number of planes that have an exceedence, the variance, and the standard deviation.

2. Samples of 20 parts from a metal punching process are selected every hour. Typically, 4% of the parts require rework. Let X deonte the number of parts in a sample of 20 that require rework.

   (a) What is the probability that X exceeds 1?

   (b) What is the probability that X exceeds 1 in at least one of the next five hours of samples?

3. Because not all airline passengers show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.

   (a) What is the probability that every passenger who shows up can take the flight?

   (b) What is the probability that the flight departs with empty seats?

4. Suppose a particular process for making wires producees an average of 2.6 flaws per 1000 meters of wire.

   (a) Find the probability that a 1000 meter length of wire has one or fewer flaws.

   (b) Find the probability that a 1000 meter length of wire has more than two flaws.

   (c) Find the probability that a 500 meter length of wire has no flaws.

5. A highway engineer who is studying the number of accidents at a busy intersection has determined that accidents occur at the rate of 2.5 per month.

   (a) Find the probability that none occurs in a given month.

   (b) Find the probability that more than one occurs in a given month.

6. Past experience has indicated that the breaking strength of the yarn used in manufacturing a particular drapery material is normally distributed and that $\sigma = 2$ psi. A random sample of nine specimens is tested, and the average breaking strength is found to be 98 psi. Find a 95% confidence interval for the true mean breaking strength.

7. The capacities (in ampere-hours) were measured for a sample of 120 batteries. The average was 178 and the standard deviation was 14.

   (a) Find a 99% confidence interval for the true mean capacity of batteries produced by this method.

   (b) Approximately how many batteries must be samples so that a 99% confidence interval will specify the mean to within ± 2 ampere-hours?

8. Based on a large sample of capacitors of a certain type, a 95% confidence interval for the mean capacitance, in μF, was computed to be $(0.213, 0.241)$. Find a 90% confidence interval for the mean capacitance of this type of capacitator.

9. A hot tub manufacturer advertises that with its heating equipment, a temperature of 100°F can be achieved in at most 15 minutes. A random sample of 30 tubs is selected, and the time necessary to achieve a 100°F temperature is determined for each tub. The sample average time is 15.8 minutes. Assume that the distribution of times of all such tubs is approximately normally distributed with a standard deviation of 2.2 minutes.

   (a) Determine appropriate null and alternate hypotheses to test the manufacturer's claim.

   (b) Compute the p-value for the hypothesis test you set up in part (a).

   (c) Given your analysis, do you believe the manufacturer's advertised claim? Why or why not?

10. Let μ be the radiation level to which a randomly selected radiation worker at a large plant is exposed during the course of a year. The Environmental Protection Agency has set the maximum safe level of exposure at 5 rem per year. If a hypothesis test is to be performed to determine whether this particular workplace is safe, which is the most appropriate alternate hypothesis: $H_A : \mu > 5$, $H_A : \mu < 5$, or $H_A : \mu \neq 5$? Justify your answer.

11. Suppose that a process used to manufacture synthetic fibers is known to produce fibers with a mean breaking strength of 50 N. A new process, which would require expensive retooling to implement, has been developed. In a sample of 1000 fibers produced by this method, the average breaking strength was 50.1 N, and the standard deviation was 1 N.

    (a) Determine appropriate null and alternate hypotheses to test the claim that the mean breaking strength of the new process is greater than the mean breaking strength of the existing process.

    (b) Compute the p-value for the hypothesis test you set up in part (a).

    (c) Given your analysis, would you advise the manufacturer to adopt the new process? (Careful, this is a bit of a trick question.)