Name: Answer Key

Math 216 Midterm 1
February 15, 2012

Problem Number	Possible Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	16	
8	16	
9	16	
10	16	
Total	100	

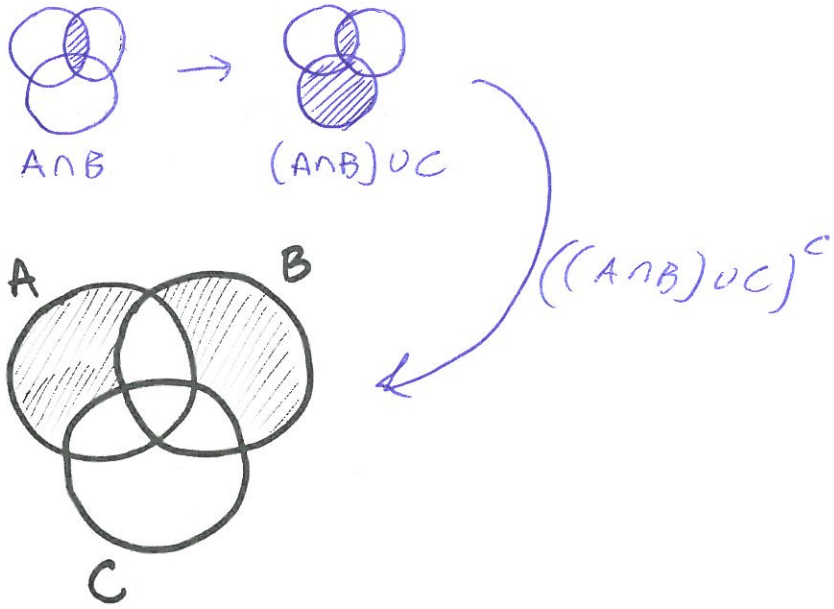
Directions—Please Read Carefully!

- Read each problem carefully and make sure to answer the specific questions asked. Some questions ask you to justify or explain your answers. You must do so to receive full credit on these questions.
- You may use a graphing or scientific calculator on this exam, as well as one 3" x 5" index card (front and back) with notes or formulas. No other aids are allowed.

Questions 1 through 6 are multiple-choice questions. Please write your answer choice for each question in the blank that precedes the question.

1. A Which of the following sets is represented by the shaded area in the Venn diagram below?

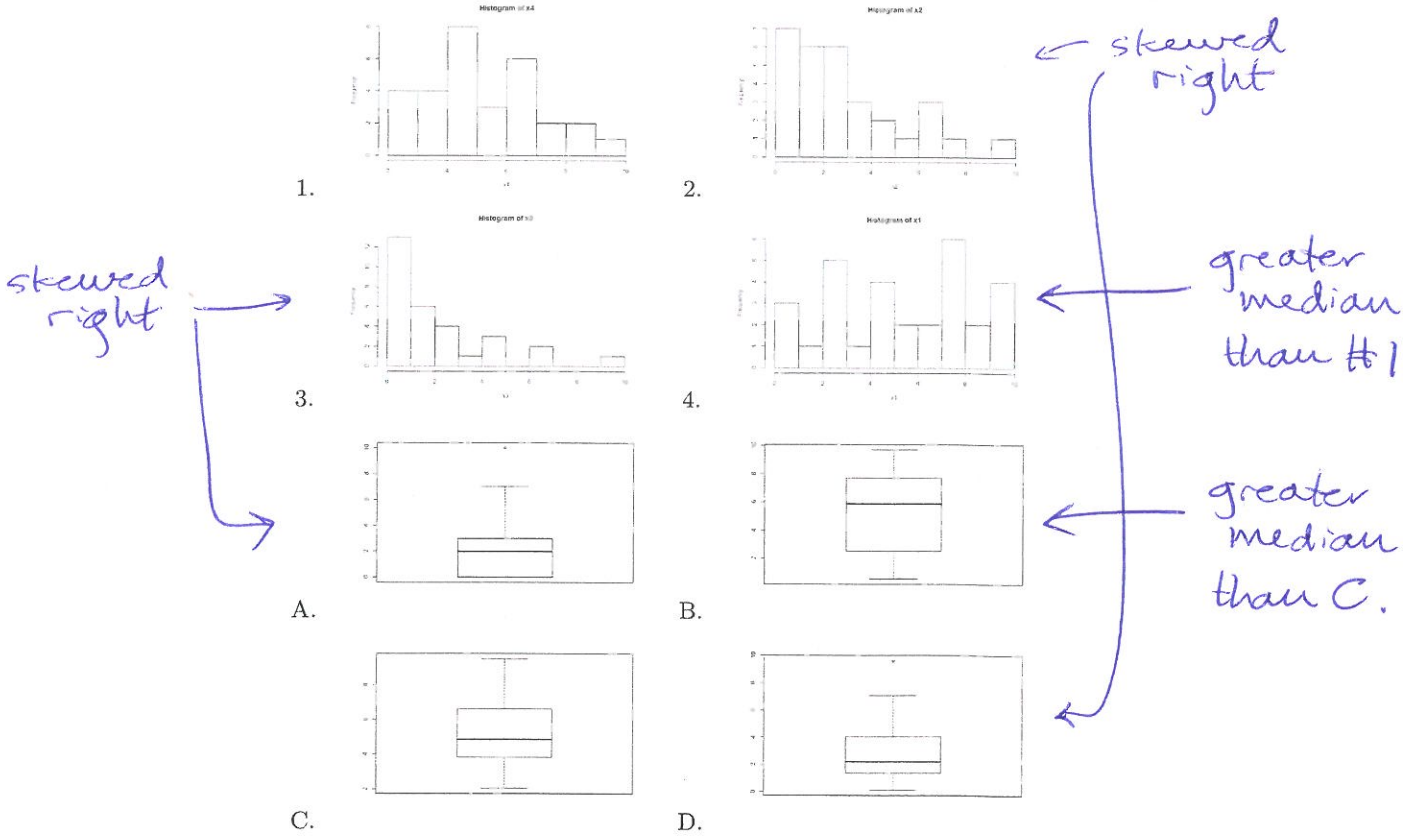
   A. $((A \cap B) \cup C)^C$
   B. $(A \cup B)^C \cap C$
   C. $(A \cup B) \cap C^C$
   D. $(A \cap B) \cup (B \cap C)$



2. B A GPS (global positioning system) unit manufacturer has developed a new model, the Pathfinder, which is supposed to help drivers navigate to their destinations more quickly than their current model, the Mapper. The manufacturer plans to recruit 50 people to test their two units under authentic driving conditions by navigating from the town center to a football stadium 10 miles away. Which of the following experimental design choices DOES NOT violate one of the four principles of experimental design?

   A. The drivers are recruited by setting up a booth at the football stadium on game day and asking for volunteers from people walking by. → selection bias/randomization problem
   B. Drivers are asked to use their own cars during the test.
   C. Of the 50 drivers, the 25 with the least experience using GPS units are given the Pathfinder to use in the test. The other 25 drivers are given the Mapper. → randomization problem
   D. Of the 50 drivers, the 25 Pathfinder users perform their test drives on Friday. The 25 Mapper users perform their test drives on Saturday. → failure to control for traffic conditions

3. C Four sets of data are displayed below, each set as a histogram and a boxplot. Which boxplot represents the same set of data as **Histogram 1**?



4. A There were 1,601 freshmen who enrolled at Vanderbilt in the fall of 2011. Of those students, 1,011 enrolled in the College of Arts & Science, 326 in the School of Engineering, 208 in Peabody, and 56 in Blair. Suppose several of these 1,601 students are selected at random. Which of the following statements is an example of the representativeness heuristic?

A. Theres a good chance that a random sample of four students will include a student from each school.

B. If a randomly selected student is taking a music class, he or she is probably a Blair student. → base rate fallacy

C. Economics is a really popular major, so it's more likely that a randomly selected student will be an Economics major than an Arts & Science student.

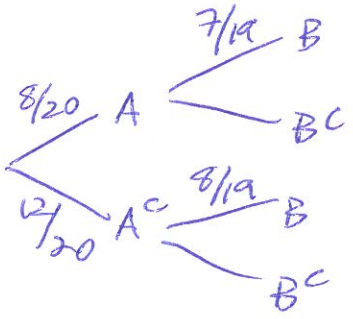
D. It's possible that a random sample of 20 students are all from Engineering.

↳ not incorrect to say

↳ conjunction fallacy

5. B From a group of 20 students, 8 of whom are computer science majors and 12 of whom are electrical engineering majors, one student is selected at random, and then a second student is selected at random from the remaining 19 students. Let A be the event that the first student is a computer science major, and let B be the event that the second student is a computer science majors. Which of the following statements is TRUE?

A. A and B are independent.
B. $P(A) = P(B)$
C. $P(A \cap B) = P(A)P(B)$
D. $P(B|A) > P(B)$



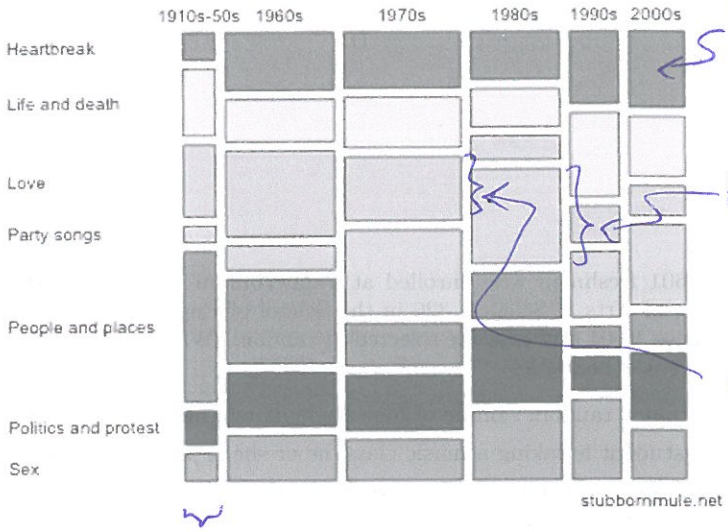
$P(B) = P(A \cap B) + P(A^c \cap B)$

$= \frac{8}{20} \cdot \frac{7}{19} + \frac{12}{20} \cdot \frac{8}{19}$

$= \frac{8}{20} = P(A)$

6. B In 2009, the UK's *Guardian* newspaper published a list of 1,000 "songs to hear before you die." The songs were categorized according to theme (heartbreak, life and death, love, and so on). The mosaic plot below shows these 1,000 songs by theme and decade of release. Based on this mosaic plot, which of the following probabilities is largest?

A. The probability that a randomly selected song was released before 1960.
B. The probability that a randomly selected song was a party song, given that it was released in the 1980s.
C. The probability that a randomly selected song was a love song, given that it was released in the 1970s.
D. The probability that a randomly selected song was released in the 2000s *and* dealt with heartbreak.



D = area of this box / area of plot

B = height of this box / height of column

C = height of this box / height of column

A = width of this column / width of plot

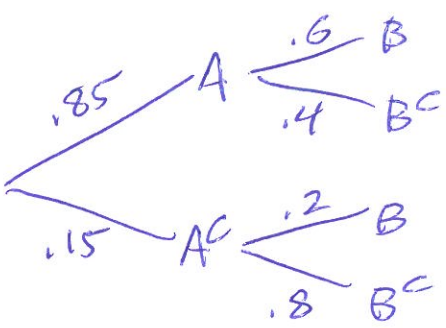
The following questions are free-response questions. For these questions, the more of your work you write down, the more easily I can grant you partial credit if your answer is incorrect.

7. An experimental search-and-rescue robot is designed to autonomously explore buildings toppled by earthquakes to look for survivors. On a controlled test run, the robot has a 60% chance of successfully locating a crash-test dummy, assuming a key navigational system remains functional. There's a 15% chance that navigational system will malfunction, in which case the chance of successfully locating the crash-test dummy drops to 20%.

   (a) What is the robot's overall probability of successfully locating the crash-test dummy?

A = nav. system works

B = finds dummy



$$P(B) = P(A \cap B) + P(A^C \cap B)$$
$$= .85(.6) + .15(.2)$$
$$= \boxed{0.54}$$

   (b) If the robot successfully locates the crash-test dummy, what is the probability that the navigational system malfunctioned?

$$P(A^C \mid B) = \frac{P(A^C \cap B)}{P(B)} = \frac{.15(.2)}{.54} = \boxed{0.0556}$$

8. A standard deck of cards consists of 13 cards (Ace, 2, 3, ..., 9, 10, Jack, Queen, King) in each of four different suits (clubs, spades, hearts, diamonds) for a total of 52 cards. Consider the following card game, played with a well-shuffled, standard deck of cards. If you draw a red card, you win nothing. If you get a spade, you win \$5. For any club, you win \$10 plus an extra \$20 for the ace of clubs.

(a) Let the random variable X be the amount of money you win in a single game. Construct the probability distribution function for X.

x	0	5	10	30
$p(x)$	$1/2$	$1/4$	$12/52$	$1/52$

(b) What is the maximum amount you would be willing to pay to play this game? *Justify your answer.*

$$E(X) = 0\left(\tfrac{1}{2}\right) + 5\left(\tfrac{1}{4}\right) + 10\left(\tfrac{12}{52}\right) + 30\left(\tfrac{1}{52}\right)$$

$$\approx \$4.13$$

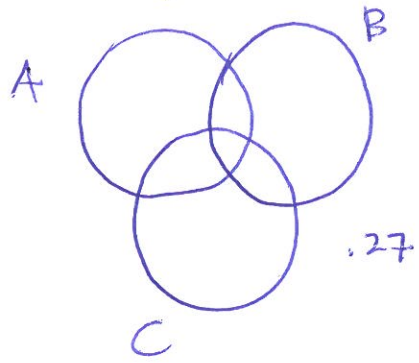
Pay any more than \$4.13 and you'll lose money in the long run.

9. An experimental rocket has three redundant guidance systems: Alpha, Beta, and Gamma. On any given flight, the probability that the Alpha system fails is 39%, that the Beta system fails is 31%, and that the Gamma system fails is 49%. The chance that both Alpha and Beta fail is 15%, that both Beta and Gamma fail is 14%, and that both Gamma and Alpha fail is 17%. There's a 27% chance that none of the three systems fail on a single flight.

(a) A flight is successful if at least one of the guidance system works. What is the probability that a flight will be successful?

A = Alpha fails
B = Beta fails
C = Gamma fails

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$



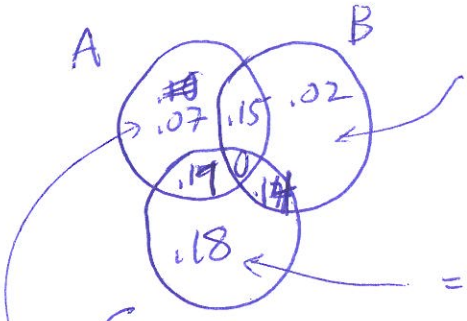
$$\Rightarrow .73 = .39 + .31 + .49 - .15 - .17 - .14 + P(A \cap B \cap C)$$

$$\Rightarrow P(A \cap B \cap C) = 0.$$

$P(\text{flight successful}) = 1 - P(A \cap B \cap C)$

$= \boxed{100\%}$

(b) What is the probability that exactly one of the systems fails on a given flight?



$= .31 - .15 - .14 = .02$

$= .49 - .17 - .14 = .18$

$= .39 - .15 - .19 = .07$

$P(\text{exactly one system fails}) = .07 + .02 + .18$

$= \boxed{27\%}$

10. The bubble chart on the next page compares total carbon dioxide (CO_2) emissions and wealth of the world's countries. Also included on the chart are per capita CO_2 emissions, population size, and region. Note the use of a logarithmic scale for both axes.

(a) Are the variables "total CO_2 emission" and "gross domestic product" associated (correlated)? If so, describe the nature of the association.

Yes, positively correlated.

(b) Identify a country that is an outlier on this chart and describe (in words) the way(s) in which it is an outlier.

Two possibilities:

① Kiribati → lowest Total CO_2 and by far the lowest Total CO_2 for countries with similar GDP per Capita

② Luxembourg → highest GDP per capita + significantly higher than other countries with similar Total CO_2

(c) Identify *two* trends or patterns that can be justified using this bubble chart.

Three possibilities:

① There seems to be a positive correlation between Total CO_2 and population size.

② There seems to be a positive correlation between GDP per Capita and CO_2 per Capita.

③ African countries generally have lower Total CO_2 and lower GDP per capita.