Resistor Tolerance

[redacted]

Math 216: Application Project
Due Date: April 24, 2007

As the production and test engineer in a large electronic component development and production plant, one of the primary tasks is to carry out statistical and error analysis on batches of sample components from the mass produced population to ensure that their values are within their stated tolerance. Today, tests are scheduled to be carried out on two carbon resistors values that were mass produced the other day.

In order to get accurate test results, it is assumed that each sample is random and consists of resistors produced at different times on the given day. The procedure used to collect the data is as follows:

> Box A contains a batch of sample resistors of a certain value (3.3 kΩ, 5% tolerance) from a large quantity that is mass produced, and box B contains a batch of sample resistors of a different value (3.3 kΩ, 10% tolerance) that is mass produced the same day. The value of each of the resistors in this sample batch and is carefully measured and recorded in a table (Appendix Table 1). It is important to note the tolerance of the resistor, which is the color of the fourth band of the resistor's color code (Watai 2007).

From the data collected, the engineer must determine if the resistors in each box fall within their stated tolerance, how the resistors of different tolerances compare, and if each box is the nominal value of the resistor (3.3 kΩ).

When performing the data analysis, it is assumed that the data follows a well-behaved distribution (single peak, relatively symmetric, and tails die rapidly), which is checked using a boxplot, histogram, and normal probability plot. It must also be assumed that the sample size is large enough to estimate the variance and use the Central Limit Theorem. The engineer is reasonably comfortable with these assumptions since the data values follow a well-behaved distribution. Lastly, it is assumed that the 5% and 10% tolerance values (3.3 kΩ ± the tolerance good

times the nominal value) represent the prediction interval for the data since the prediction of the individual values of the resistors is considered, unlike the confidence interval which focuses on the true mean of the response.

Table 2 shows the data that was calculated to create the parallel boxplots for the 5% and 10% tolerance resistors, which are then shown in Figure 1.

	5%	10%
LIF	3.18	3.35
Q1	3.27	3.41
Median	3.3	3.43
Q3	3.33	3.45
UIF	3.42	3.51

Table 2

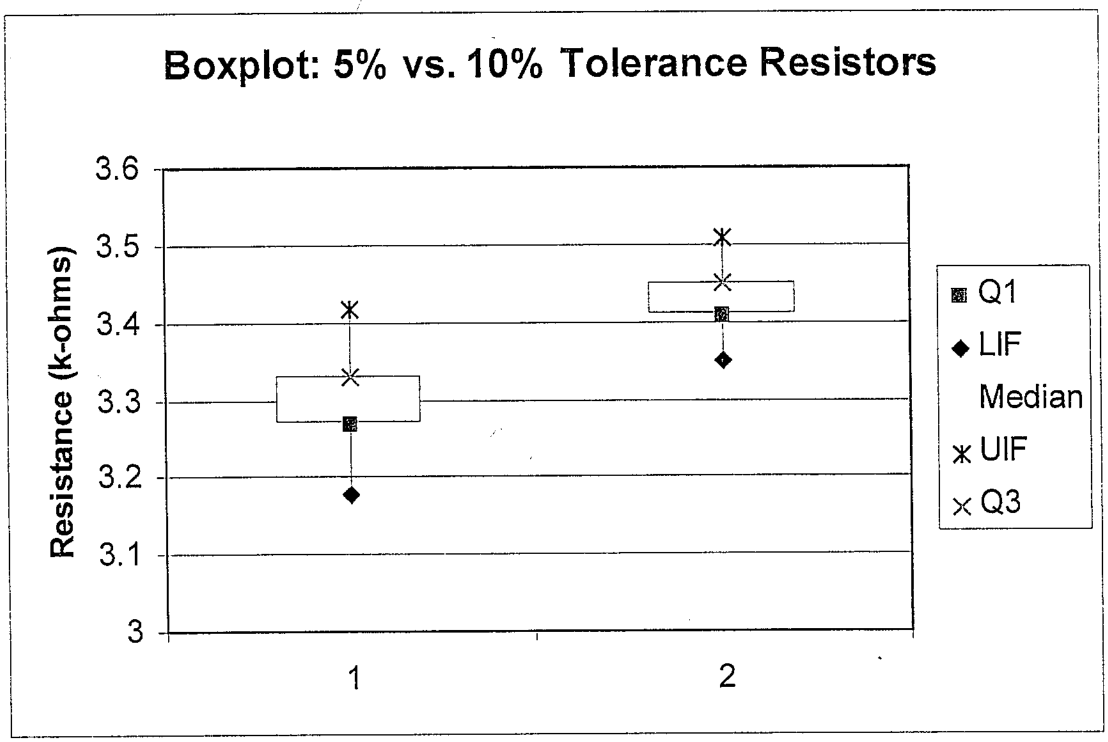

Figure 1
(1 represents the 5% tolerance, and 2 represents the 10% tolerance.)

From the parallel boxplots for the data, it can be seen that the 5% tolerance resistors are centered around a value of 3.3 kΩ while the 10% tolerance resistors are centered around a value of 3.43 kΩ. All of the data for the 5% tolerance resistors is contained within the inner fences. The 10% tolerance resistors have one mild outlier with a value of 3.32, which is lower than the lower inner fence and above the lower outer fence for stated data. It can also be seen that most of the 10% tolerance resistors have greater values than the 5% tolerance resistors have. interesting

Even though the 10% tolerance resistors contain an outlier, the 5% tolerance resistors have greater overall variation in their data. This can be seen in that the interquartile range for the 5% tolerance resistors is greater than the interquartile range of the 10% tolerance resistors. Both of the resistor's data are fairly symmetric.

Table 3 shows the data that was calculated to create the histograms for the 5% and 10% tolerance resistors, which are then shown in Figure 2.

Bin	*Frequency (5%)*	*Frequency (10%)*
3.1	0	0
3.15	0	0
3.2	2	0
3.25	5	0
3.3	21	0
3.35	13	1
3.4	7	9
3.45	2	31
3.5	0	7
3.55	0	2
3.6	0	0

Table 3

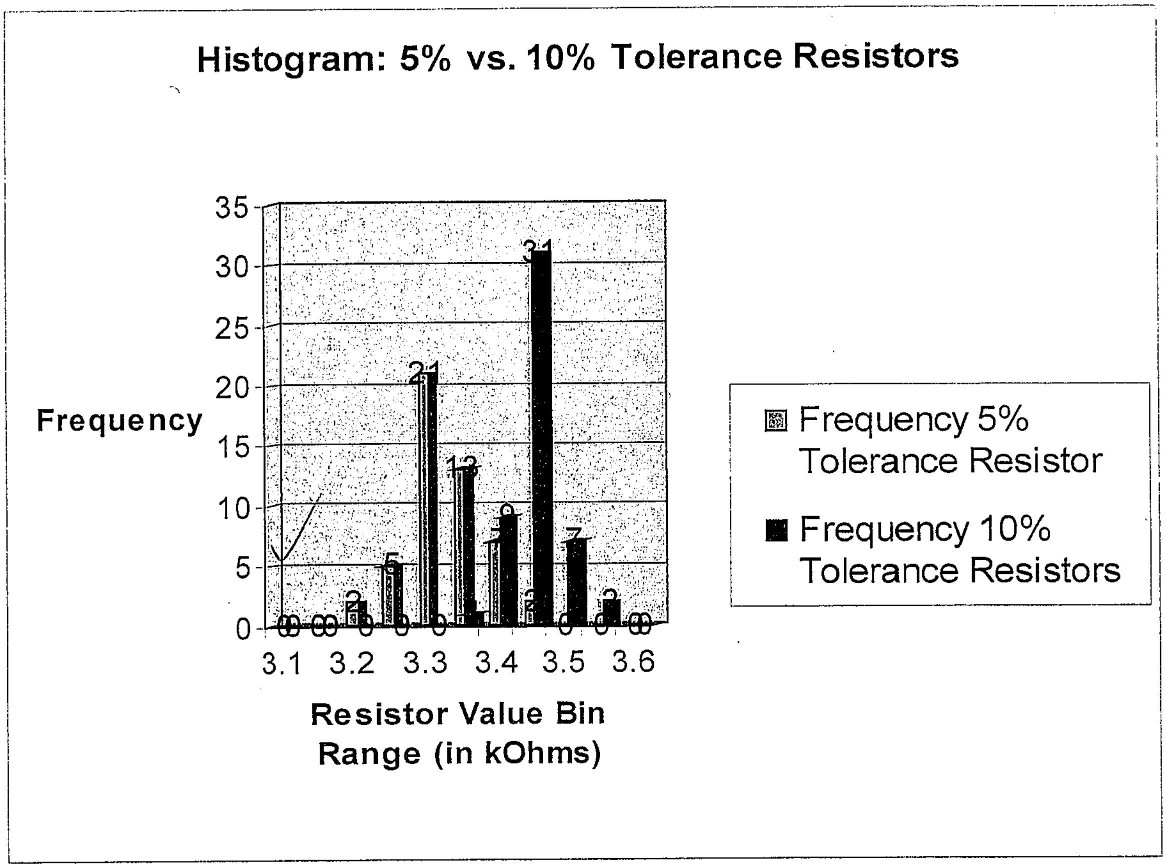

Figure 2

From the histograms for the data, it can be seen that the largest number of the 5% tolerance resistors fall around a value of 3.3 kΩ while most of the 10% tolerance resistors fall around a value of 3.45 kΩ. The 5% tolerance resistors have greater overall variation in their data and more spread out across the bins. These results are similar to those found in the parallel boxplot analysis. Also, it can be seen that the data follows a well-behaved distribution with a single peak, tails that die rapidly, and is relatively symmetric. However, unlike in the boxplots, outliers cannot be determined from the histograms above.

Table 4 shows the data that was calculated to create the normal probability plots for a small portion of the 5% and 10% tolerance resistor data, which are then shown in Figure 3.

i	xi (5%)	xi (10%)	Pi (Percentile)	zi (Normal Quantile)
1	3.36	3.32	0.071428571	-1.465
2	3.28	3.43	0.214285714	-0.79
3	3.29	3.44	0.357142857	-0.365
4	3.2	3.49	0.5	0
5	3.32	3.5	0.642857143	0.365
6	3.32	3.49	0.785714286	0.79
7	3.41	3.43	0.928571429	1.465

Table 4

Why only 7 data points?

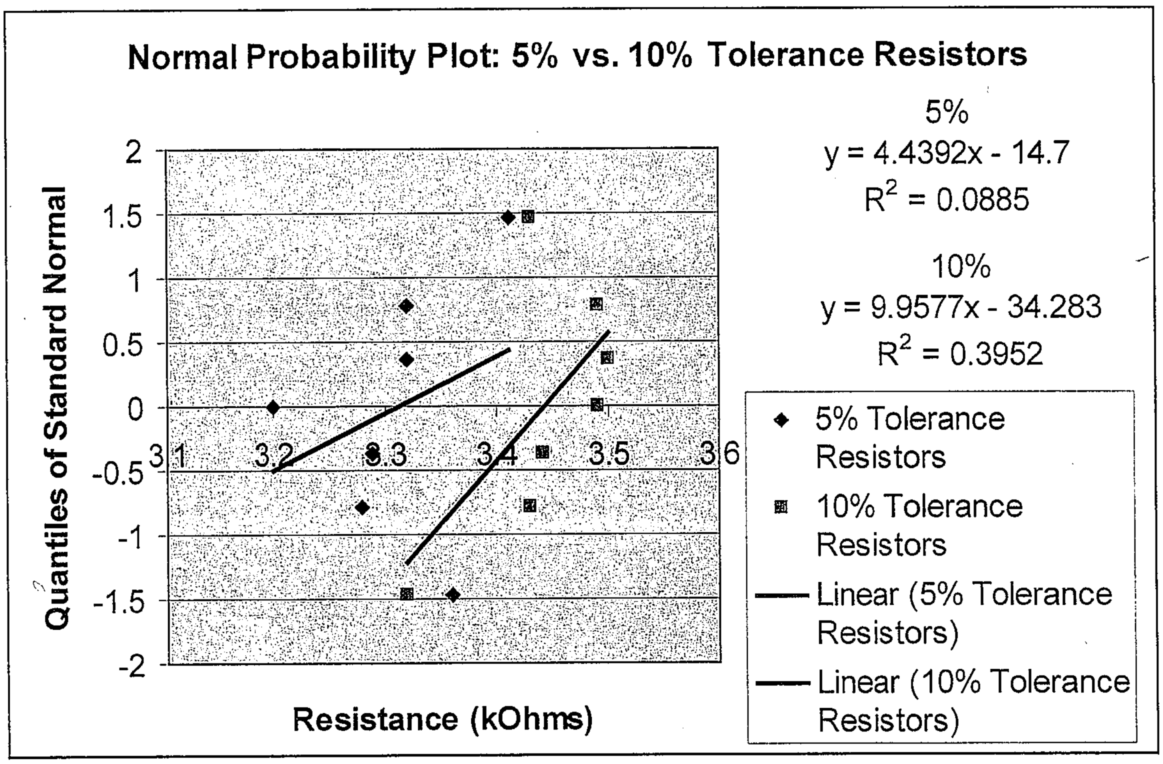

Figure 3

Because the data is random, the first seven resistor values in both Box A and Box B were used to construct a normal probability plot for the 5% and 10% resistors, respectively. From Figure 3, it can be seen that $R^2 = 0.089$ for the 5% and resistors and $R^2 = 0.395$ for the 10% resistors. It is once again shown that the 5% resistors have greater variability by the smaller R^2 value. Though, both R^2 values are very far from one and show that the data is not well approximated by a linear model. However, only seven points were used to create this figure, and why Assume?

since fifty values are used in the following calculations, we can still have some confidence that a normal probability plot for all the points may follow a linear model with less variability. Therefore, we will still assume that the data can be reasonably modeled by a normal distribution.

The confidence intervals for the data are found using the equation:

$(y_{bar} - t_{n-1,\alpha/2}\,(s\,/\,sqrt\,n),\ y_{bar} + t_{n-1,\alpha/2}\,(s\,/\,sqrt\,n))$

where n-1 is the degrees of freedom. Equation 1

After calculating the values for $\bar{Y}$, the t-distribution, and the sample standard deviation, and using a 95% confidence level, the confidence interval found for the mean of the 5% tolerance resistors was $3.286 < \mu < 3.314$. The confidence interval for the mean of the 10% tolerance resistors was found to be $3.421 < \mu < 3.442$. We are reasonably comfortable that the true mean resistance for the 5% tolerance resistors is actually about 3.3 kΩ since this value is in the confidence interval. We should not think that the true mean resistance for the 10% tolerance resistors is the stated 3.3 kΩ since this value falls outside of the 95% confidence interval for the data.

In addition a two-sided hypothesis test was conducted for the 5% and 10% tolerance resistors with a 95% confidence level. The null hypothesis was Ho: μ=3.3 kΩ and the alternative hypothesis was Ha: $\mu \neq$3.3 kΩ. The test statistic for the 5% resistor was calculated to be 0.0284 and the t-distribution value for $t_{49,0.025} = 2.01$. Because $.0284 < 2.01$, the null hypothesis should not be rejected and it is reasonable for the mean to be 3.3 kΩ. The test statistic for the 10% resistor was calculated to be 25.114 and the t-distribution value for $t_{49,0.025} = 2.01$. Because $25.114 > 2.01$, the null hypothesis should be rejected and it not is reasonable for the mean to be 3.3 kΩ. The mean should therefore fall within the confidence interval found above to be $3.421 < \mu < 3.442$.

From the analysis of the collected data, the engineer determined that although all the resistors measured in Box A fell within the stated 5% tolerance (3.135 to 3.465 kΩ), and all the resistors in Box B also fell within the stated 10% tolerance (2.970 to 3.630 kΩ). However, it was found that the true mean of Box B was not 3.3 kΩ using a two-sided hypothesis test and then a confidence interval to determine the plausible mean values, which were higher than the nominal value. If the plant wants to maintain a mean of 3.3 kΩ, the 10% resistor production process must be adjusted. The mean of 3.3 kΩ for Box A was not rejected with the hypothesis test and was found to also be contained in the confidence interval. The 5% resistor production process should not be changed. In addition, it was determined that the 10% resistors do not have a greater variance than the 5% resistors. This means that the plant ~~does not need to keep~~ the variability as small for the 10% resistors, thus decreasing production costs. good! Lastly, the importance of sample size was seen in the normal probability plot; the sample size must be large enough to assume that the data is well-behaved, and this can also be checked using a boxplot or histogram. Using these statistical tests, a company can quantify the success of their production process and use the results to implement process changes.

Works Cited

Watai, Lason L. Challenge #1B: Basic Statistics and Error Analysis in Engineering. 5 March 2007. Vanderbilt University: Electrical Engineering and Computer Science. Accessed 10 April 2007 < http://eecs.vanderbilt.edu/courses/ee213/challenge1b.htm>.

Appendix

Box A			Box B			average A	average B
color-code	3.3 kOhms		color-code	3.3 kOhms		3.3002	3.4316
tolerance	5%		tolerance	10%		var A	var B
measurements	% error	xa^2	measurements	% error	xb^2	0.002471	0.001373
3.36	-1.81818	11.2896	3.32	-0.60606	11.0224	stddev A	stddev B
3.28	0.606061	10.7584	3.43	-3.93939	11.7649	0.049713	0.037053
3.29	0.30303	10.8241	3.44	-4.24242	11.8336	test stat	test stat
3.2	3.030303	10.24	3.49	-5.75758	12.1801	0.028448	25.11434
3.32	-0.60606	11.0224	3.5	-6.06061	12.25	3.286069	3.421068
3.32	-0.60606	11.0224	3.49	-5.75758	12.1801	3.314331	3.442132
3.41	-3.33333	11.6281	3.43	-3.93939	11.7649	confidence intervals	
3.31	-0.30303	10.9561	3.38	-2.42424	11.4244		
3.36	-1.81818	11.2896	3.4	-3.0303	11.56		
3.28	0.606061	10.7584	3.41	-3.33333	11.6281		
3.26	1.212121	10.6276	3.44	-4.24242	11.8336		
3.38	-2.42424	11.4244	3.51	-6.36364	12.3201		
3.27	0.909091	10.6929	3.5	-6.06061	12.25		
3.28	0.606061	10.7584	3.43	-3.93939	11.7649		
3.28	0.606061	10.7584	3.47	-5.15152	12.0409		
3.31	-0.30303	10.9561	3.51	-6.36364	12.3201		
3.37	-2.12121	11.3569	3.44	-4.24242	11.8336		
3.3	0	10.89	3.42	-3.63636	11.6964		
3.3	0	10.89	3.4	-3.0303	11.56		
3.28	0.606061	10.7584	3.45	-4.54545	11.9025		
3.21	2.727273	10.3041	3.44	-4.24242	11.8336		
3.33	-0.90909	11.0889	3.43	-3.93939	11.7649		
3.33	-0.90909	11.0889	3.42	-3.63636	11.6964		
3.3	0	10.89	3.41	-3.33333	11.6281		
3.26	1.212121	10.6276	3.41	-3.33333	11.6281		
3.33	-0.90909	11.0889	3.4	-3.0303	11.56		
3.28	0.606061	10.7584	3.44	-4.24242	11.8336		
3.29	0.30303	10.8241	3.43	-3.93939	11.7649		
3.25	1.515152	10.5625	3.42	-3.63636	11.6964		
3.23	2.121212	10.4329	3.45	-4.54545	11.9025		
3.32	-0.60606	11.0224	3.48	-5.45455	12.1104		
3.26	1.212121	10.6276	3.4	-3.0303	11.56		
3.19	3.333333	10.1761	3.45	-4.54545	11.9025		
3.31	-0.30303	10.9561	3.47	-5.15152	12.0409		
3.28	0.606061	10.7584	3.45	-4.54545	11.9025		
3.26	1.212121	10.6276	3.45	-4.54545	11.9025		
3.36	-1.81818	11.2896	3.42	-3.63636	11.6964		
3.34	-1.21212	11.1556	3.4	-3.0303	11.56		
3.27	0.909091	10.6929	3.39	-2.72727	11.4921		
3.3	0	10.89	3.38	-2.42424	11.4244		
3.25	1.515152	10.5625	3.42	-3.63636	11.6964		
3.32	-0.60606	11.0224	3.41	-3.33333	11.6281		
3.26	1.212121	10.6276	3.45	-4.54545	11.9025		

3.36	-1.81818	11.2896	3.41	-3.33333	11.6281
3.25	1.515152	10.5625	3.38	-2.42424	11.4244
3.28	0.606061	10.7584	3.42	-3.63636	11.6964
3.32	-0.60606	11.0224	3.42	-3.63636	11.6964
3.38	-2.42424	11.4244	3.41	-3.33333	11.6281
3.42	-3.63636	11.6964	3.42	-3.63636	11.6964
3.31	-0.30303	10.9561	3.44	-4.24242	11.8336

sum xa	sum xa^2	sum xb	sum xb^2
165.01	544.6871	171.58	588.8612
(sum xa)^2		(sum xb)^2	
27228.3		29439.7	

Table 1