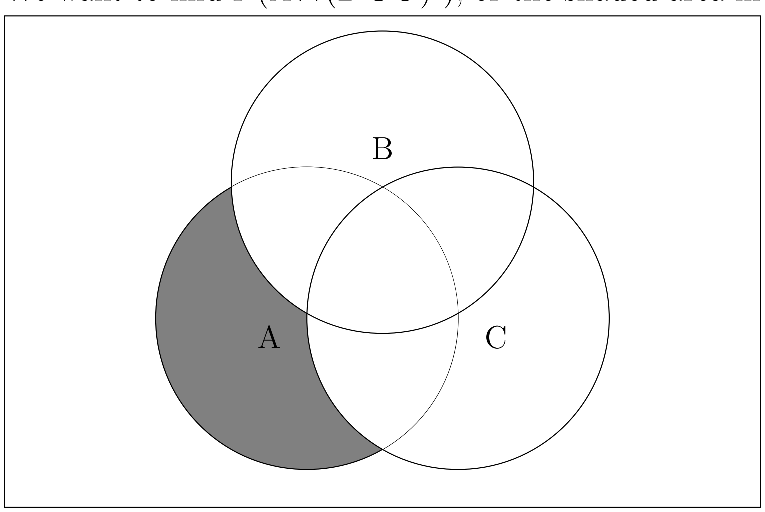
Math 216 Spring 2012
Problem Set 2 Answer Key

1. The biggest problem with Professor Z's strategy is the **sampling bias**. Engineering students are disproportionately male, so the simulation would be biased toward boy-boy and boy-girl combinations. [**6pts**]

2. Answers will vary. A correct answer will identify a lurking variable that would explain both habitual coffee drinking and lower uterine cancer rates. Examples: Not smoking (Non-smokers might be more likely to drink coffee, since they are both stimulants, and not smoking might contribute to lower cancer rates); Income (High wage earning women might be more likely to drink several cups of coffee during a work day, higher income might also contribute to better health care...) [**6pts**]

3. [**20pts**]
   $A = \{12, 13, 14, ...\}$
   $B = \{0, 1, ..., 14, 15\}$
   $D = \{8, 9, 10, 11\}$
   (a) $(A \cup D)^c = \{0, 1, 2, 3, 4, 5, 6, 7\}$
   (b) $A \cap B \cap D = \emptyset$ (empty set)
   (c) $B^c \cap D = \emptyset$
   (d) $A \cup (B \cap D) = \{8, 9, 10, 11, 12, ...\}$

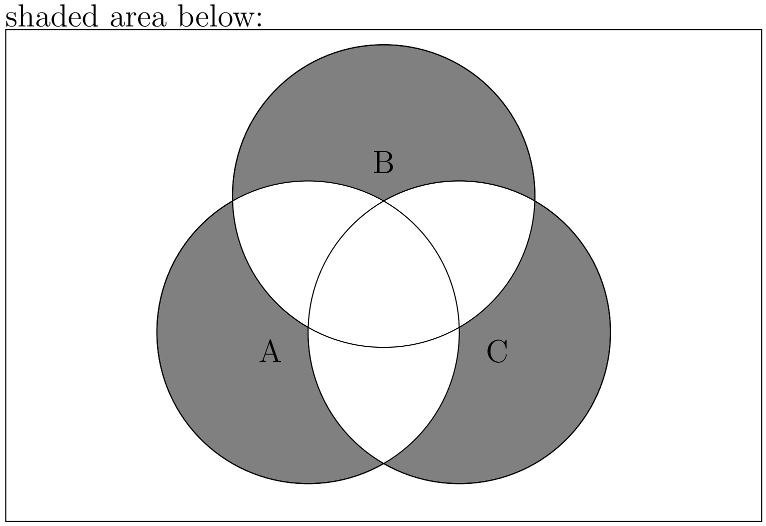
4. [**10pts**]
   (a) We want to find $P(A \cap (B \cup C)^c)$, or the shaded area in the venn diagram below:



This is just the probability that the next purchaser will request A, B, or C minus the probability that they request B or C. In other words,

$$\begin{aligned} P(A \cap (B \cup C)^c) &= P(A \cup B \cup C) - P(B \cup C) \\ &= 0.98 - 0.95 = \mathbf{0.03} \end{aligned}$$

(b) We want to find $P(A \cap (B \cup C)^c) + P(B \cap (A \cup C)^c) + P(C \cap (A \cup B)^c)$, or the shaded area below:



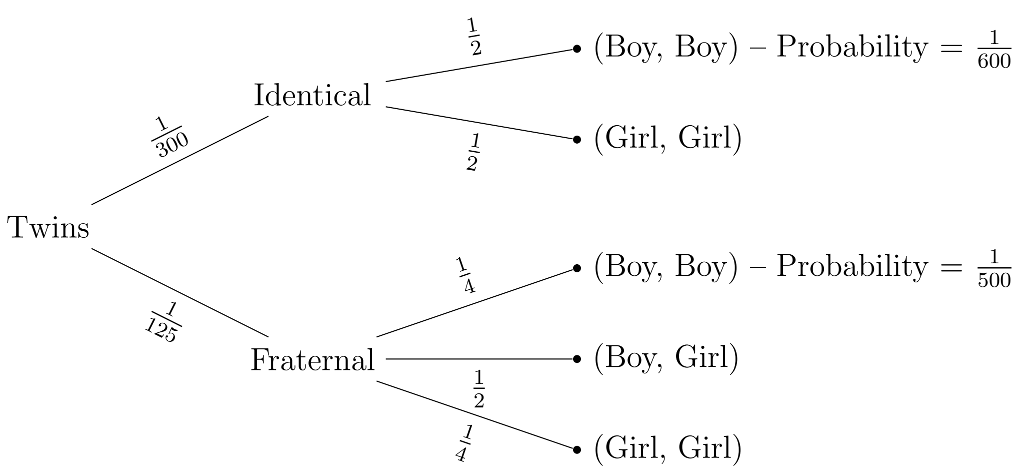
We can find these in the same way we found the answer for part (a).

$$P(A \cap (B \cup C)^c) = 0.03$$
$$P(B \cap (A \cup C)^c) = P(A \cup B \cup C) - P(A \cup C) = 0.98 - 0.90 = 0.08$$
$$P(C \cap (A \cup B)^c) = P(A \cup B \cup C) - P(A \cup B) = 0.98 - 0.85 = 0.13$$

So

$$\begin{aligned} P(A \cap (B \cup C)^c) + P(B \cap (A \cup C)^c) + P(C \cap (A \cup B)^c) &= 0.03 + 0.08 + 0.13 \\ &= \mathbf{0.24} \end{aligned}$$

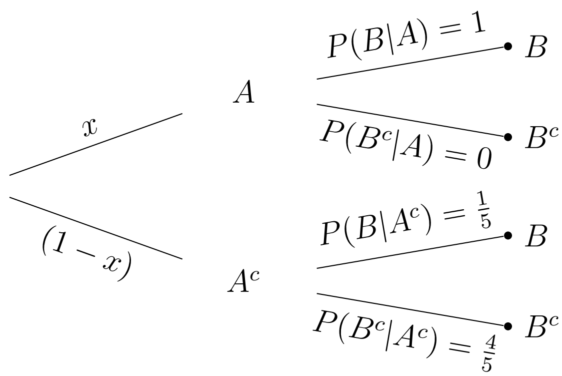
5. **[10pts]** We can construct the following tree diagram:



We are interested in the conditional probability of identical twin boys given twin boys.

$$P(Identical \cap (Boy, Boy) | (Boy, Boy)) = \frac{\frac{1}{600}}{\frac{1}{600} + \frac{1}{500}} = \frac{\mathbf{5}}{\mathbf{11}} \approx \mathbf{0.45}$$

6. **[10pts]** Define the following events:
A = Sam knows the answer
B = Sam answers correctly
Then we can construct the following tree diagram. Since there are five answer choices for each question, we assume if Sam guesses, he gets the correct answer with probability 1/5. Also, we assume Sam always answers correctly when he knows the answer, and incorrectly when he does not know. Define x to be $P(A)$.

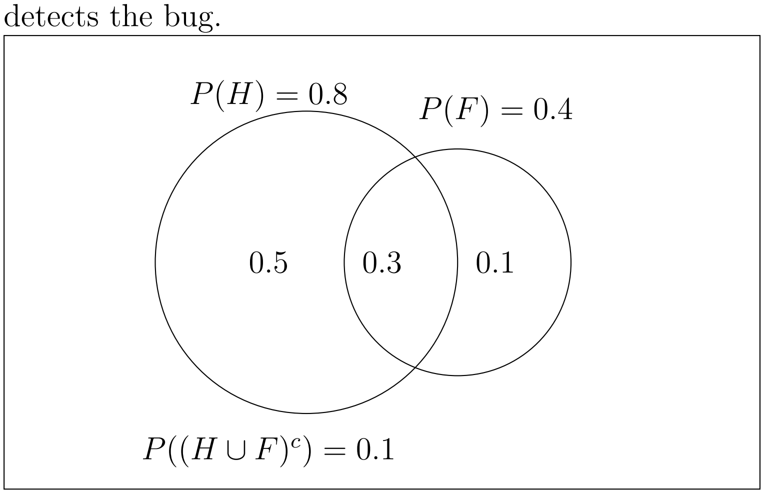


We are given that $P(A|B) = 0.92$. Bayes' Rule gives us:

$$0.92 = P(A|B) = \frac{P(B|A) * P(A)}{P(B)} = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|A^c) * P(A^c)}$$
$$= \frac{1 * x}{1 * x + (1 - x) * \frac{1}{5}}$$

Solve to get $\mathbf{x \approx 0.70}$.

7. **[15pts]** Construct a Venn Diagram. Let H = Phineas detects the bug and F = Ferb detects the bug.

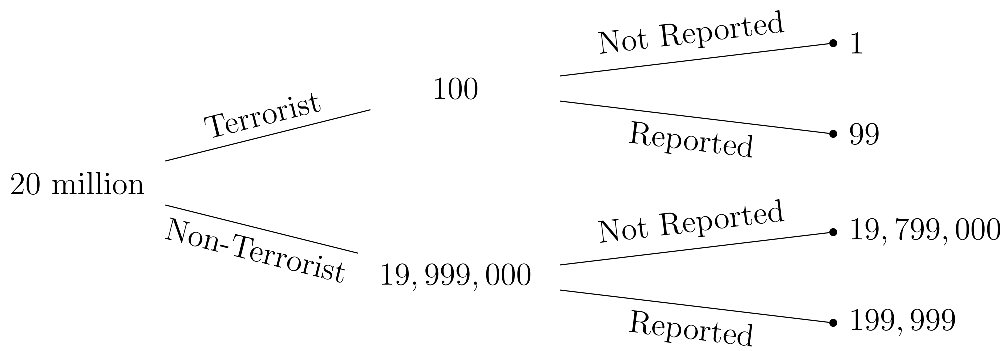


(a) $P((H \cup F)^c) = 1 - (0.5 + 0.3 + 0.1) = \mathbf{0.1}$

(b) To show dependence, show that $P(H) \neq P(H|F)$ or $P(F) \neq P(F|H)$. I will show the former:
$P(H|F) = 0.3/0.4 = 0.75$, but $P(H) = 0.8$. Therefore the events H and F are dependent.

(c) Example: Phineas and Ferb might have received the same training, so they are likely to detect the same type of bugs. *Note: Again, dependence in this case does not imply causation.*

8. [**15pts**] As usual, construct a tree diagram!



(a) $99 + 199,999 = \mathbf{200,098}$

(b) $99/200,098 = \mathbf{0.000495}$

(c) No right answer here, just curious what you all thought.

9. [**8pts**]

```
> USA <- subset(earthquakes2000, COUNTRY == 'USA' & EQ_MAG_MW != 'NA',
  select = c(COUNTRY,LATITUDE,LONGITUDE,EQ_MAG_MW))
> radius <- sqrt(USA$EQ_MAG_MW/pi)
> symbols( USA$LONGITUDE, USA$LATITUDE, circles = radius, inches =.12)
```