Math 216
Problem Set 3

1. Suppose a game is played by rolling two six-sided dice and subtracting the smaller number from the larger number rolled. So, for instance, if you roll a 3 and a 5, the resulting score is $5 - 3 = 2$. If both dice show the same number, take the score to be 0. Let the random variable X be your score on one roll of the dice.

    (a) What is the probability distribution function for X?

    (b) What is $E(X)$?

    (c) What is $\text{Var}(X)$?

2. A company has five warehouses, only two of which have a particular product in stock. A salesperson calls the five warehouses in random order until a warehouse with the product is reached. Let the random variable X be the number of calls made by the salesperson.

    (a) What is the probability distribution function for X?

    (b) What is $E(X)$?

3. Suppose that a random variable X is the time (in hours) to failure of a newly charged batter. (Failure here is defined to be the moment at which the battery can no longer supply enough energy to operate a certain applicance.) Suppose X has probability density function

$$f(x) = \frac{2}{(x+1)^3}$$

    for $x \geq 0$ and $f(x) = 0$ for all $x < 0$.

    (a) Confirm that f is a valid probability density function. That is, confirm that $f(x) \geq 0$ for all x and that $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

    (b) What is the probability that one of these batteries will fail in the first five hours?

    (c) What is the probability that one of these batteries will last at least five hours?

4. Suppose the hour hand of a broken clock is stuck pointing at 12 and that the minute hand can be spun at will. Consider spinning the minute hand and then measuring the angle θ it makes with the hour hand when it comes to a stop. Let θ be measured so that it always lies between 0 and 180 degrees. Assume that the minute hand spins "fairly," in that after spinning, it is no more likely to point in any particular direction than another.

    (a) What is the probability density function for the random variable θ? Be sure to specific where this function takes the value 0.

    (b) Use your answer to part (a) to find the probability that the minute hand will end up between 45 and 90 degrees from the hour hand.

5. A circular piston head is designed to slide smoothly within a cylinder. However, there is some variability in the piston head and cylinder sizes about their specified dimensions. Suppose that the random variable X measures the radius of a piston head, and that it has an expected value of 30 mm and a standard deviation of 0.05 mm. Also suppose that the random variable Y measures the inside radius of the cylinder, with an expected value of 30.25 mm and a standard deviation of 0.06 mm.

(a) What is the expected value of the "gap" between the piston head and the cylinder?

(b) What is the standard deviation of this "gap"?

(c) If the random variable Z measures the "gap" and is normally distributed with mean and standard deviation equal to the values you found in parts (a) and (b), what is the probability that Z will be negative?

6. The thickness of glass sheets produced by a certain process are normally distributed with a mean of 3 mm and a standard deviation of 0.12 mm.

    (a) What is the probability that a glass sheet is thicker than 3.2 mm?

    (b) What is the probability that a glass sheet is thinner than 2.7 mm?

    (c) What is the value of c for which there is a 99% probability that a glass sheet has a thickness within the interval $[3 - c, 3 + c]$?

7. The weights of bags filled by a machine are normally distributed with a standard deviation of 0.05 kg and a mean that can be set by the operator. At what level should the mean be set if it is required that only 2% of the bags weight less than 10 kg?

8. Exercise 3.20 in your textbook, page 137.