Math 194
Unit 6 Practice

1. Find a unit vector in the direction of each of the following vectors.

   (a) $\begin{bmatrix} -30 \\ 40 \end{bmatrix}$

   (b) $\begin{bmatrix} 7/4 \\ 1/2 \\ 1 \end{bmatrix}$

2. Determine whether each pair of vectors is orthogonal.

   (a) $\begin{bmatrix} 8 \\ -5 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ 2 \\ -5 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 1 \\ -2 \\ 6 \end{bmatrix}$

3. True or False: For an $n \times m$ matrix A, vectors in the row space of A are orthogonal to vectors in the null space of A.

4. True or False: If the distance from $\mathbf{x}$ to $\mathbf{y}$ equals the distance from $\mathbf{x}$ to $-\mathbf{y}$, then $\mathbf{x}$ and $\mathbf{y}$ are orthogonal.

5. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}$.

   (a) Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$.

   (b) Express $\mathbf{x}$ as a linear combination of the $\mathbf{u}$'s.

6. For each of the following, find the orthogonal projection of $\mathbf{y}$ onto the subspace spanned by the $\mathbf{u}$'s.

   (a) $\mathbf{y} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$

   (b) $\mathbf{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

7. For each of the following data sets, find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the given data points.

   (a) $(0, 1), (1, 1), (2, 2), (3, 2)$

   (b) $(-1, 0), (0, 1), (1, 2), (2, 4)$

8. For each of the following sets, determine if the set is a subspace of $\mathbb{P}_4$. *Justify your answers.*
   (a) The set of polynomials in $\mathbb{P}_4$ of degree 3.
   (b) The set of polynomials p in $\mathbb{P}_4$ such that $p(1) = 0$.
   (c) The set of polynomials in $\mathbb{P}_4$ having at least one real root.

9. For each of the following sets, determine if the set spans $\mathbb{P}_3$. *Justify your answers.*
   (a) $\{x^2 - 3x, 2x^2 - 9x, 3x^2 + x + 1, 10x\}$
   (b) $\{x^2 + x, 0, x\}$
   (c) $\{2x^2 + 1, x^2 + x, 3x + 4\}$
   (d) $\{x^2 + 2x - 3, 4x^2 - 5x + 6\}$

10. Let $p_1(x) = 1 + 2x + x^2$, $p_2(x) = 2 + 9x$, $p_3(x) = 3 + 3x + 4x^2$, and $q(x) = 2 + 17x - 3x^2$.
    (a) Show that $\{p_1, p_2, p_3\}$ is a basis for $\mathbb{P}_3$.
    (b) Express q as a linear combination of p_1, p_2, and p_3.

11. Let U and V be subspaces of the (abstract) vector space W. The intersection of U and V is defined to be the set $U \cap V$ consisting of all vectors in W that are both U and V. Is $U \cap V$ a subspace of W? *Justify your answer.*

12. *Challenge:* A certain experiment produces the data $(1, 1.8), (2, 2.7), (3, 3.4), (4, 3.8), (5, 3.9)$. Find the least-squares fit of these points by a function of the form $y = \beta_1 x + \beta_2 x^2$.

1. (a) $\begin{bmatrix} -.6 \\ .8 \end{bmatrix}$

   (b) $\begin{bmatrix} 7/\sqrt{69} \\ 2/\sqrt{69} \\ 4/\sqrt{69} \end{bmatrix}$

2. (a) Not orthogonal

   (b) Orthogonal

3. True

4. True

5. (a) Hint: If a set of vectors is orthogonal, then the set must also be linearly independent.

   (b) $\mathbf{x} = \frac{5}{2}\mathbf{u}_1 - \frac{3}{2}\mathbf{u}_2 + 2\mathbf{u}_3$

6. (a) $\hat{\mathbf{y}} = \begin{bmatrix} 10/3 \\ 2/3 \\ 8/3 \end{bmatrix}$

   (b) $\hat{\mathbf{y}} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$

7. (a) $y = .9 + .4x$

   (b) $y = 1.1 + 1.3x$

8. (a) No

   (b) Yes

   (c) No

9. (a) Yes

   (b) No

   (c) Yes

   (d) No

10. (a) Hint: You must show that these functions span P_3 and that they are linearly independent.

    (b) $q = p_1 + 2p_2 - p_3$

11. Yes

12. $y = 1.76x - .20x^2$