

Math 194
Unit 3 Practice

1. Determine whether each of the following sets is a subspace. Justify your answers.

- (a) $\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 1\}$
- (b) $\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 0\}$
- (c) $\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 \leq x_2 \leq x_3\}$
- (d) $\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 = -x_3\}$

2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 6 & 9 & 6 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & 4 & 9 & 1 & 2 \end{bmatrix}$.

- (a) Find a basis for the row space of A .
- (b) Find a basis for the column space of A .
- (c) Find a basis for the null space of A .
- (d) What is the rank of A ?

3. Suppose that A is a 5×4 matrix and that $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ is in the null space of A . Write \mathbf{a}_4 as a linear combination of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 .

4. Do the following vectors form a basis for \mathbb{R}^4 ? $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \\ -8 \end{bmatrix}$

5. Find a basis for the subspace of \mathbb{R}^3 defined by the equation $2x_1 + 3x_2 + x_3 = 0$.

6. True or False: The row space and column space of an invertible matrix are the same.

7. True or False: If A and B are $n \times n$ matrices with the same row space, then A and B have the same column space.

8. True or False: If E is an $n \times n$ elementary matrix and A is an $n \times n$ matrix, then A and EA have the same row space.

9. For each of the following, find the largest possible value for the rank of A and the smallest possible value for the dimension of the null space of A .

- (a) A is 5×3 .
- (b) A is 3×5 .

(c) A is 4×4 .

10. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. Which of the following vectors is in the nullspace of A ?

(a) $\begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ -1 \\ 3 \\ 2 \end{bmatrix}$

(e) None of the above vectors is in the nullspace of A .

11. Suppose A is a 3×4 matrix with rank 3.

(a) Do the rows of A form a basis for the row space of A ? Justify your answer.

(b) Do the columns of A form a basis for the column space of A ? Justify your answer.

1. (a) Not a subspace
- (b) Subspace
- (c) Not a subspace
- (d) Subspace

2. (a) $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

- (b) $\begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 9 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$

- (c) $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

- (d) 3

3. $\mathbf{a}_4 = -\frac{1}{4}\mathbf{a}_1 - \frac{1}{2}\mathbf{a}_2 - \frac{3}{4}\mathbf{a}_3$

4. Yes

5. $\begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

6. True

7. False

8. True

9. (a) 3, 0

- (b) 3, 2

- (c) 4, 0

10. (c)

11. (a) Yes

- (b) No