Math 194
Unit 2 Practice

1. Let $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 1 \\ -3 & 3 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, and $G = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$. Which of the following can be computed?
   (a) $A + 2B$
   (b) $A - B^T$
   (c) $4D - 3C^T$
   (d) $D - D^T$
   (e) $G + (2F)^T$
   (f) $(7A - B) + E$

2. Using the matrices from Question 1, which of the following can be computed?
   (a) CD
   (b) AE
   (c) FG
   (d) $B^T F$
   (e) BB^T
   (f) GE

3. Using the matrices from Question 1, find each of the following.
   (a) $b_{3,2}$
   (b) $\mathbf{a}_2$
   (c) $(FG)_{1,2}$
   (d) $F\mathbf{b}_1$

4. Suppose that an $n \times n$ matrix A satisfies the equation $A^2 - 2A + I = 0$. Find values for α and β such that $A^3 = \alpha A + \beta I$.

5. A square matrix A is said to be *idempotent* if $A^2 = A$. Show that if A is idempotent, then so is $I - A$.

6. True or False: If $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions to the matrix equation $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_1 \neq \mathbf{x}_2$, then A is invertible.

7. True or False: The product of two elementary matrices is always an elementary matrix.

8. Let $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$.

   (a) Write A^{-1} as the product of two elementary matrices.

   (b) Write A as the product of two elementary matrices.

9. True or False: Every invertible matrix can be expressed as the product of elementary matrices.

10. Determine whether each set of vectors is linearly independent.

    (a) $\mathbf{v}_1 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 10 \\ 2 \end{bmatrix}$

    (b) $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$

    (c) $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

    (d) $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -6 \\ 2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}$

11. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a set of linearly independent vectors in $\mathbb{R}^3$.

    (a) Remove any one vector from the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Is the resulting set of vectors linearly independent?

    (b) Add any vector from $\mathbb{R}^3$ to the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Is the resulting set of vectors linearly independent?

12. True or False: If A is a 4×3 matrix with linearly independent columns, then the matrix equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$ in $\mathbb{R}^4$.

13. True or False: If $\mathbf{v}_3$ is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

14. Suppose that $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ are linearly independent vectors in $\mathbb{R}^4$ and that A is an invertible 4×4 matrix. Let

$$\mathbf{y}_1 = A\mathbf{x}_1, \qquad \mathbf{y}_2 = A\mathbf{x}_2, \qquad \mathbf{y}_3 = A\mathbf{x}_3.$$

    Is the set $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ linearly independent, as well? Justify your answer.

15. An company has three divisions: widgets, doohickeys, and macguffins. View the company as an open economy described by the accompanying table, where input is in dollars needed for \$1.00 of output.

	Input Consumed Per Unit of Output		
Purchased From	Widgets	Doohickeys	Macguffins
Widgets	\$0.50	\$0	\$0.25
Doohickeys	\$0.20	\$0.80	\$0.10
Macguffins	\$1	\$0.40	\$0

(a) Suppose that the customers of this company have a demand for \$30 worth of widgets, \$20 worth of doohickeys, and \$40 worth of macguffins. What production levels are needed for each of the three divisions to meet this outside demand?

(b) Suppose that the customers increase their demand for doohickeys by \$1. By what amounts should production increase for each of the three divisions to meet this increased demand?

1. (a), (c), (d), (e)
2. All except (f)
3. (a) 0

   (b) $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

   (c) 8

   (d) $\begin{bmatrix} -5 \\ 2 \\ 16 \end{bmatrix}$
4. $\alpha = 3$, $\beta = -2$
5. $(I - A)^2 = I - 2A + A^2 = I - 2A + A = I - A$
6. False
7. False
8. (a) $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
9. True
10. (a) linearly independent

    (b) linearly dependent

    (c) linearly independent

    (d) linearly dependent
11. (a) Yes

    (b) No
12. False
13. False
14. Yes
15. (a) \$880 worth of production for widgets, \$1800 for doohickeys, and \$1640 for macguffins

    (b) \$10 increase for widgets, \$25 increase for doohickeys, and \$20 increase for macguffins