Math 194
Unit 1 Practice

1. Find the polynomial of degree 2 whose graph goes through the points $(1, -1)$, $(2, 3)$, and $(3, 13)$.

2. Solve the following system of linear equations.

   $x_4 + 2x_5 - x_6 = 2$

   $x_1 + 2x_2 + x_5 - x_6 = 0$

   $x_1 + 2x_2 + 2x_3 - x_5 + x_6 = 2$

3. Consider two $n \times m$ matrices in reduced row-echelon form to be of the same type if they contain the same number of leading 1s in the same positions. For example, $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are of the same type. How many types of 3×2 matrices in reduce row-echelon form are there?

4. Is there a sequence of elementary row operations that transforms $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ into $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$? Justify your answer.

5. Let A be a 4×4 matrix and let $\mathbf{b}$ and $\mathbf{c}$ be two vectors in $\mathbb{R}^4$. Suppose you know that the matrix equation $A\mathbf{x} = \mathbf{b}$ has no solution. What can you say about the number of solutions of the system $A\mathbf{x} = \mathbf{c}$?

6. Is the vector $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$? Justify your answer.

7. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}$. Which of the following sets has the same span as the set of all four vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\, \mathbf{v}_4\}$? (Justify your answers.)

   (a) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

   (b) $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$

   (c) $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

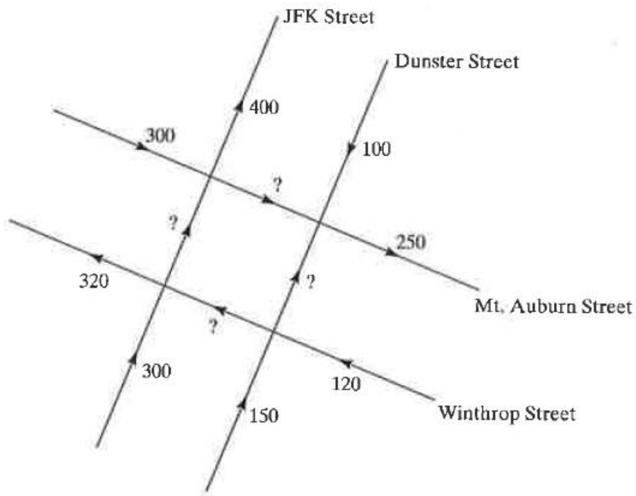
   (d) $\{\mathbf{v}_3, \mathbf{v}_4\}$

8. True or False: If A and B are any two 3×3 matrices whose reduced row-echelon forms have two pivots, then A and B are row equivalent.

9. True or False: If a vector $\mathbf{v}$ in $\mathbb{R}^4$ is a linear combination of the vectors $\mathbf{u}$ and $\mathbf{w}$ and if A is a 5×4 matrix, then $A\mathbf{v}$ must be a linear combination of $A\mathbf{u}$ and $A\mathbf{w}$.

10. While waiting for your bus on a busy city street, you notice that (a) all of the vehicles passing your position are cars and trucks, (b) three out of every four trucks that pass you are followed by a car, and (c) only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

11. The figure below shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour. Find the highest and lowest possible traffic volume for each of the street segments indicated by question marks.



12. Consider an economy with three sectors: mining, electricity, and robotics. Mining sells 30% of its output to electricity, 50% to robotics, and retains the rest. Electricity sells 80% of its output to mining, 10% to robotics, and retains the rest. Robotics sells 40% to mining, 40% to electricity, and retains the rest.

    Find a set of equilibrium prices (that is, prices at which each sector's income matches its expenses) when the price for the robotics output is $100 million.

1. $f(x) = 3x^2 - 5x + 1$
2. $x_1 = -2x_2 - x_5 + x_6$
   $x_3 = x_5 - x_6 + 1$
   $x_4 = -2x_5 + x_6 + 2$
   x_2, x_5, and x_6 free
3. 4 types
4. There is not.
5. It has infinitely many solutions or none.
6. Yes, it is.
7. (b), and (c)
8. False
9. True
10. 4/19
11. Winthrop: between 120 and 270 cars per hour; JFK: between 100 and 250; Mt. Auburn: between 0 and 150; Dunster: between 0 and 150
12. \$141.7 million for mining, \$91.7 million for electricity, and \$100 million for robotics