Math 194
Problem Set 9

For Question 1, you may use Wolfram Alpha or some other computational tool, but only to row reduce matrices. All other work must be shown to receive full credit.

1. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$.

   (a) List the eigenvalues of A, repeated according to their multiplicities.

   (b) For each of the eigenvalues you found in part (a), find a basis for the corresponding eigenspace.

2. Consider the following linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.

   - L_1 is a reflection about the line $y = 3x$.
   - L_2 is a projection onto the x-axis.
   - L_3 is a horizontal shear by a factor of 4.
   - L_4 is a scale by 3 in the x-direction and by 5 in the y-direction.

   Let $A_1, \ldots, A_4$ be the corresponding standard matrices. Which standard matrix does not have two linearly independent eigenvectors? Justify your answer.

3. Suppose that λ is an eigenvalue of the matrix A with eigenvector $\mathbf{x}$.

   (a) Find an eigenvalue for the matrix A^2. Justify your answer.

   (b) Assume A is invertible. Find an eigenvalue for A^{-1}. (Hint: Simplify $A^{-1}A\mathbf{x}$ in two different ways.)

4. In the town of Desert Bluffs, shopkeepers hire security guards to protect their shops from burglars. Let s_k be the number of shopkeepers in Desert Bluffs k months from now, b_k be the number of burglars in Desert Bluffs k months from now, and g_k be the number of security guards in Desert Bluffs k months from now. Suppose that the following relationships hold.

$$\begin{array}{rcrcrcr} s_{k+1} & = & 1.1s_k & - & 0.1b_k & - & 0.05g_k \\ b_{k+1} & = & 0.3s_k & + & 0.95b_k & - & 0.2g_k \\ g_{k+1} & = & 0.2s_k & + & 0.4b_k & + & 0.7g_k \end{array}$$

   This results in a discrete dynamical system with transition matrix $A = \begin{bmatrix} 1.1 & -0.1 & -0.05 \\ 0.3 & 0.95 & -0.2 \\ 0.2 & 0.4 & 0.7 \end{bmatrix}$.

   (a) Why is it reasonable that the $1,2$ entry in A is negative?

   (b) Why is it reasonable that the $3,2$ entry in A is positive?

   (c) Why is it reasonable that the $1,3$ entry in A is negative?

   (d) If the security guards were to become more effective at capturing burglars, which entry in A would change? Would the new entry be less than or greater than the current entry?

5. Answer the following questions about the shopkeeper-burglar-guard system described above.

(a) Over time, do you expect the number of shopkeepers, burglars, and guards in Desert Bluffs to grow, decline, or stay the same? If it grows or declines, at what rate? *Justify your answer.*

(b) In the long run, what will be the ratio of security guards to shopkeepers in Desert Bluffs?

6. Suppose you have a population described by the discrete dynamical system $\mathbf{x}_{k+1} = A\mathbf{x}_k$ for $k = 0, 1, 2, \ldots$ and for some 3×3 matrix A. If A has eigenvalues $\lambda_1 \approx 0.988$, $\lambda_2 \approx 0.878 + 0.434i$, and $\lambda_3 \approx 0.878 - 0.434i$, then what can you say about the longterm population growth or decline? Does it grow, decline, or stay the same? Justify your answer.