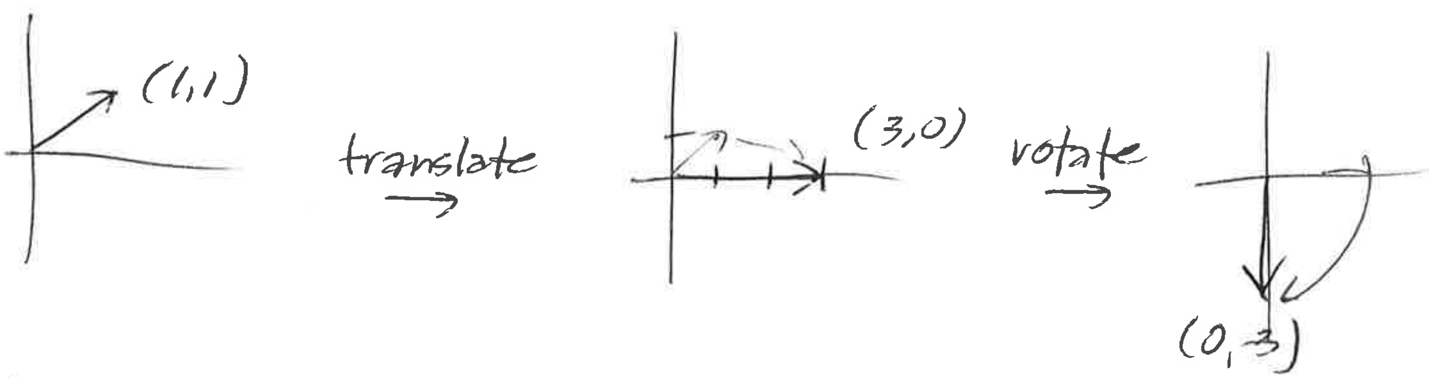
PROBLEM SET 8

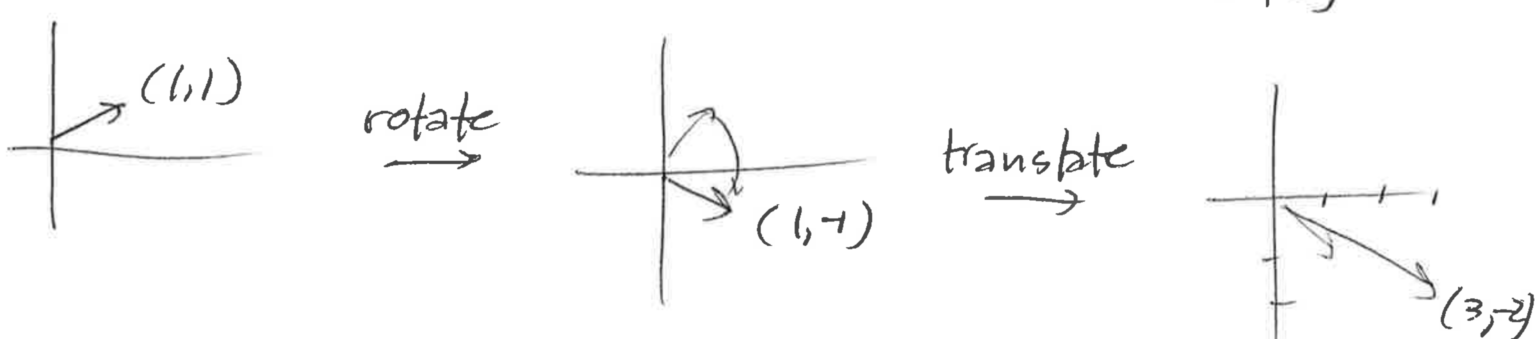
① (a) Translation by 2 units in the x-direction and -1 units in the y-direction, <u>then</u> rotation by 90° clockwise about the origin.

(b) vice versa

(c) $AB = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ $BA = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ Nope.

you can also follow a particular vector. For instance, $\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.





$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$ but $L\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

② (a) $A\vec{v} = \begin{bmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 42 \\ -30 \\ 24 \end{bmatrix}$

not multiples so $\vec{v}$ is NOT an eigenvector

(b) $A\vec{v} = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ -2 \end{bmatrix} = -2\vec{v}$

$\Rightarrow$ $\vec{v}$ is an eigenvector with eigenvalue $\lambda = -2$.

③ (a) $A\vec{x} = \lambda\vec{x}$

$(A - \lambda I)\vec{x} = \vec{0}$

$A = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$ $\lambda = 10$ $\Rightarrow$ $\left[\begin{array}{cc|c} -6 & -2 & 0 \\ -3 & -1 & 0 \end{array}\right]$

$\longrightarrow \left[\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \longrightarrow$ $3x_1 + x_2 = 0$, x_2 free

$\longrightarrow$ $x_1 = -\frac{1}{3}x_2$, x_2 free $\longrightarrow$ $\vec{x} = \begin{bmatrix} -1/3\, x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1/3 \\ 1 \end{bmatrix}$

Basis $= \left\{ \begin{bmatrix} -1/3 \\ 1 \end{bmatrix} \right\}$ or any scalar multiple.

~~(b)~~ $\cancel{\left[(A - \lambda I) \mid \vec{0} \right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & -2 & -3 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]}$

$\longrightarrow$ $x_1 = -2x_2 + 3x_3$ $\longrightarrow$ $\vec{x} =$

x_2, x_3 free

(b) $(A-\lambda I)\vec{x}=\vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & -2 & -3 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\longrightarrow$ $x_1 = -2x_2 - 3x_3$ $\longrightarrow$ $\vec{x} = \begin{bmatrix} -2x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix}$

x_2, x_3 free

$$= x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

basis =

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

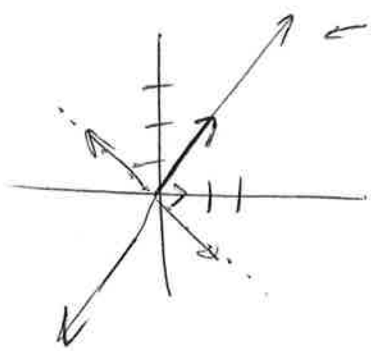
4) $A^{45}\vec{x} = \underbrace{A(A(A(A(\cdots Ax))))\cdots)}_{45 \text{ copies}}$.

Since $A\vec{x} = \lambda\vec{x} = -\vec{x}$

we have that $A^{45}\vec{x} = (-1)^{45}\vec{x} = -\vec{x} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

5) $2x_2 = 3x_1$

$x_2 = \frac{3}{2}x_1$



← reflect across this line

$\underline{\lambda = 1}$: Anything on the line will remain fixed.

$\underline{\lambda = -1}$: Anything $\vec{x}$ on the line $3x_2 = -2x_1$ (the perpendicular) will map to $-\vec{x}$