

Math 194
The Invertible Matrix Theorem (Unit 5 Version)

Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A , the statements are either all true or all false.

1. A is an invertible matrix.
2. A is row equivalent to the $n \times n$ identity matrix.
3. A has n pivot positions.
4. A has a pivot in each column.
5. A has a pivot in each row.
6. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
7. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each \mathbf{b} in \mathbb{R}^n .
8. The equation $A\mathbf{x} = \mathbf{b}$ has exactly one solution for each \mathbf{b} in \mathbb{R}^n .
9. The columns of A span \mathbb{R}^n .
10. The columns of A are linearly independent.
11. The columns of A form a basis for \mathbb{R}^n .
12. The nullspace of A equals $\{\mathbf{0}\}$.
13. The dimension of the nullspace of A is 0.
14. The column space of A is \mathbb{R}^n .
15. The dimension of the column space of A is n .
16. The row space of A is \mathbb{R}^n .
17. The dimension of the row space of A is n .
18. The rank of A is n .
19. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto.
20. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
21. The determinant of A is not zero.
22. The number 0 is not an eigenvalue of A .