Math 194
Problem Set 8

1. Suppose $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

   (a) Let T be the transformation with standard matrix AB. What 2D transformation is performed by T when it acts on the homogeneous coordinates of a point (x, y)?

   (b) Let L be the transformation with standard matrix BA. What 2D transformation is performed by T when it acts on the homogeneous coordinates of a point (x, y)?

   (c) Is the transformation performed by T the same as the transformation performed by L? Justify your answer.

2. For each of the following, determine if $\mathbf{v}$ is an eigenvector of A. If so, find the eigenvalue.

   (a) $\mathbf{v} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{bmatrix}$

   (b) $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$

3. For each of the following, find a basis for the eigenspace corresponding to the listed eigenvalue.

   (a) $A = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$, $\lambda = 10$

   (b) $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$, $\lambda = 3$

4. Suppose the 2×2 matrix A has an eigenvalue $\lambda = -1$ with associated eigenvector $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. What is $A^{45}\mathbf{x}$?

5. Let A be the standard matrix for the linear transformation that reflects vectors in $\mathbb{R}^2$ across the line $2x_2 = 3x_1$. Without finding the matrix A, find both eigenvalues of A and describe their eigenspaces.