Math 194
Problem Set 6

1. Consider an open economy described by the table below. Here's how to read this table: The first columns says that for each dollar of output from agriculture, the sector needs 27 cents of input from itself, 15 cents of input from manufacturing, 6 cents of input from trade, 27 cents of input from services, and 23 cents of input from energy.

	Agriculture	Manufacturing	Trade	Services	Energy
Agriculture	$0.27	$0.39	$0.03	$0.02	$0.23
Manufacturing	$0.15	$0.15	$0.10	$0.01	$0.22
Trade	$0.06	$0.07	$0.36	$0.15	$0.35
Services	$0.27	$0.08	$0.07	$0.41	$0.09
Energy	$0.23	$0.19	$0.36	$0.24	$0.10

   (a) Find the production levels that will satisfy the following external demand (units in millions of dollars): $1.2 from agriculture, $3.4 from manufacturing, $2.7 from trade, $4.3 from services, and $2.9 from energy.

   (b) Use the matrix $(I - C)^{-1}$, where C is the consumption matrix for this system, to determine which *three* sectors will be affected most if the external demand for services doubles.

2. Which of the following sets are subspaces of R^2? Justify your answers.

   (a) $\{\mathbf{x} \in \mathbb{R}^2 \mid x_1 + x_2 = 0\}$

   (b) $\{\mathbf{x} \in \mathbb{R}^2 \mid x_1 x_2 = 0\}$

   (c) $\{\mathbf{x} \in \mathbb{R}^2 \mid |x_1| = |x_2|\}$

   (d) $\{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 = x_2^2\}$

3. Find a basis for the subspace spanned by the vectors $\begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 9 \\ -6 \\ 12 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -4 \\ 5 \\ -3 \\ 7 \end{bmatrix}$. What is the dimension of the subspace?

4. Let $A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{bmatrix}$.

   (a) Find the null space of A.

   (b) Find the column space of A.

   (c) Find the rank of A.

5. (a) Suppose A is a 4×4 matrix whose column space is not equal to $\mathbb{R}^4$. What can you say about the null space of A?

(b) If A is a 5×5 matrix and the null space of A is equal to the subspace $\{\mathbf{0}\}$, what can you say about solutions of equations of the form $A\mathbf{x} = \mathbf{b}$ for $\mathbf{b}$ in $\mathbb{R}^5$?

(c) What can you say about the null space of the matrix B if B is a 5×4 matrix with linearly independent columns?

(d) If possible, construct a 3×4 matrix A such that the dimension of the null space of A is 2 and the dimension of the column space of A is also 2.