# Problem Set 4

① (a) $2A = \begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix}$ $\quad (2A)^T = \begin{bmatrix} 6 & -4 & 2 \\ 2 & 0 & 4 \\ 8 & 2 & 4 \end{bmatrix}$

$3B = \begin{bmatrix} 3 & 0 & 6 \\ -9 & 3 & 3 \\ 6 & -12 & 3 \end{bmatrix}$ $\quad (2A)^T - 3B = \begin{bmatrix} 3 & -4 & -4 \\ 11 & -3 & 1 \\ 2 & 14 & 1 \end{bmatrix}$

(b) $AB = \begin{bmatrix} 8 & -15 & 11 \\ 0 & -4 & -3 \\ -1 & -6 & 6 \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} 11 & 11 & 21 \\ -5 & 0 & 6 \\ 1 & 5 & 10 \end{bmatrix}$

$AB - A^2 = \begin{bmatrix} -3 & -26 & -10 \\ 5 & -4 & 3 \\ -2 & -11 & -4 \end{bmatrix}$

② (a) CBC (= can't be computed)

(b) $2 \times 3$

(c) CBC

(d) $2 \times 3$

(e) $2 \times 3$

(f) $2 \times 1$

(g) $3 \times 3$

(h) CBC

(i) $3 \times 2$

(j) CBC

③ $\vec{x}_k = \begin{bmatrix} x_{k,1} \\ x_{k,2} \end{bmatrix}$ → % of viewers watching Channel 2 in month k
→ " " " " " 5 " "

$$\vec{x}_{k+1} = \begin{bmatrix} .85 & .10 \\ .15 & .90 \end{bmatrix} \vec{x}_k$$

← P (transition matrix)

(a) $\vec{x}_0 = \begin{bmatrix} 20\,000 \\ 10\,000 \end{bmatrix}$ $\quad \vec{x}_1 = \begin{bmatrix} .85 & .10 \\ .15 & .90 \end{bmatrix} \begin{bmatrix} 20000 \\ 10000 \end{bmatrix} = \begin{bmatrix} 18000 \\ 12000 \end{bmatrix}$

month 1

$\vec{x}_2 = \begin{bmatrix} .85 & .10 \\ .15 & .90 \end{bmatrix} \begin{bmatrix} 18000 \\ 12000 \end{bmatrix} = \begin{bmatrix} 16500 \\ 13500 \end{bmatrix}$ ← month 2

$\vec{x}_3 = \begin{bmatrix} .85 & .10 \\ .15 & .90 \end{bmatrix} \begin{bmatrix} 16500 \\ 13\,500 \end{bmatrix} = \begin{bmatrix} 15375 \\ 14625 \end{bmatrix}$ ← month 3

(b) $P^3 = \begin{bmatrix} .653125 & .23125 \\ .346875 & .76875 \end{bmatrix}$

④ tur $\longrightarrow \begin{bmatrix} 19 \\ 20 \\ 17 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 & 3 \\ 3 & 1 & 1 \\ 15 & 5 & 1 \end{bmatrix} \begin{bmatrix} 19 \\ 20 \\ 17 \end{bmatrix} = \begin{bmatrix} 170 \\ 94 \\ 402 \end{bmatrix}$

$\xrightarrow[\text{MOD 26}]{} \begin{bmatrix} 14 \\ 16 \\ 12 \end{bmatrix} \longrightarrow$ OQM

ing $\rightarrow \begin{bmatrix} 8 \\ 13 \\ 6 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 & 3 \\ 3 & 1 & 1 \\ 15 & 5 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 13 \\ 6 \end{bmatrix} = \begin{bmatrix} 91 \\ 43 \\ 191 \end{bmatrix} \xrightarrow[\text{MOD 26}]{} \begin{bmatrix} 13 \\ 17 \\ 9 \end{bmatrix} \longrightarrow$ NRJ

$\boxed{\text{OQMNRJ}}$

(5) Since $\vec{x}=\vec{0}$ satisfies $A\vec{x}=\vec{0}$, we know the equation has at least one solution. Does it have exactly one solution or infinitely many solutions?

Note that $\vec{0} = 2\vec{a}_1 + \vec{a}_2 - 4\vec{a}_3 = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$

$$= A \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}.$$

Thus $\vec{x} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$ also satisfies $A\vec{x}=\vec{0}$. Since $A\vec{x}=\vec{0}$ has at least <u>two</u> solutions, it must have $\boxed{\text{infinitely many solutions}}$ since there are no options between "exactly one" and "infinitely many."