Math 194
Problem Set 4

1. If $A = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$, compute each of the following by hand (without using Wolfram Alpha or any other similar tool).
   (a) $(2A)^T - 3B$
   (b) $AB - A^2$

2. Let $A = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 0 \\ -1 & 7 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 6 & -1 \\ 3 & 9 & 8 \\ 1 & 0 & 5 \end{bmatrix}$, and $D = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. For each of the following, find the size of the resulting matrix, if it can be computed. Identify the expressions that can't be computed.
   (a) $A + B$
   (b) AB
   (c) BA
   (d) BC
   (e) ABC
   (f) BD
   (g) $C + C^T$
   (h) $B + B^T$
   (i) $(AB)^T$
   (j) $A^T B^T$

3. Two local television channels, Channel 2 and Channel 5, compete for viewers for the 6 o'clock local news. Each viewer is generally loyal to a particular channel, but every month about 15% of Channel 2's viewers get annoyed at the weatherman and start watching Channel 5. Similarly, every month about 10% of Channel 5's viewers get tired of the Channel 5 mascot, an overly cheerful penguin, and switch to Channel 2.
   (a) Assuming Channel 2 currently has 20,000 viewers and Channel 5 has 10,000 viewers and the trends described above continue, how many viewers will each channel have one month from now? Two months? Three months?
   (b) Assuming the trends described above continue, construct the transition matrix that describes how viewers switch or stay from Channels 2 and 5 *over a three-month period.*

4. Encipher the plaintext `turing` using the Hill cipher with key $\begin{bmatrix} 1 & 5 & 3 \\ 3 & 1 & 1 \\ 15 & 5 & 1 \end{bmatrix}$, assuming a 26-letter alphabet. Note that since this key is a 3×3 matrix, you will apply it to three letters at a time from the plaintext.

5. Let A be a 3×3 matrix and suppose that
$$2\mathbf{a}_1 + \mathbf{a}_2 - 4\mathbf{a}_3 = \mathbf{0}.$$
How many solutions will the equation $A\mathbf{x} = \mathbf{0}$ have? Justify your answer.