Math 194
Clicker Questions

Unit 1: Systems of Linear Equations

1. Suppose you have a system of linear equations consisting of three equations in two unknowns. How many solutions are possible for such a system? *Mark all that apply.*

   (a) No solutions.

   (b) Exactly one solution.

   (c) Infinitely many solutions.

2. Consider the following system of linear equations.

$$\begin{array}{rcrcrcr} x & + & 3y & + & z & = & 3 \\ 2x & + & 2y & + & 4z & = & -4 \\ 3x & + & y & - & z & = & 9 \end{array}$$

   The corresponding augmented matrix is

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 2 & 4 & -4 \\ 3 & 1 & -1 & 9 \end{bmatrix}.$$

   Which of the following augmented matrices correspond to a system of linear equations with the same solution set? *Mark all that apply.*

   (a) Multiply the first row by 2:

$$\begin{bmatrix} 2 & 6 & 2 & 6 \\ 2 & 2 & 4 & -4 \\ 3 & 1 & -1 & 9 \end{bmatrix}$$

   (b) Multiply the first column by 2:

$$\begin{bmatrix} 2 & 3 & 1 & 3 \\ 4 & 2 & 4 & -4 \\ 6 & 1 & -1 & 9 \end{bmatrix}$$

   (c) Add the first row to the third row:

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 2 & 4 & -4 \\ 4 & 4 & 0 & 12 \end{bmatrix}$$

   (d) Interchange the first and second rows:

$$\begin{bmatrix} 2 & 2 & 4 & -4 \\ 1 & 3 & 1 & 3 \\ 3 & 1 & -1 & 9 \end{bmatrix}$$

   (e) Interchange the first and second columns:

$$\begin{bmatrix} 3 & 1 & 1 & 3 \\ 2 & 2 & 4 & -4 \\ 1 & 3 & -1 & 9 \end{bmatrix}$$

3. Suppose the augmented matrix for a system of three linear equations in four unknowns has three pivot columns. True or False: The system must be consistent.

4. Suppose the augmented matrix for a system of three linear equations in three unknowns has a pivot in each of the first three columns. True or False: The system has exactly one solution.

5. Suppose you row-reduce the augmented matrix for a system of four linear equations in three unknowns and end up with a row of all zeros. How many solutions does the system have? *Mark all that are possible.*

   (a) No solutions
   (b) Exactly one solution
   (c) Infinitely many solutions

6. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Geometrically, what is the set of *all* linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2\}$?

   (a) A point
   (b) A line
   (c) All of $\mathcal{R}^2$

7. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following is *not* a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$
   (d) All of these are linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

8. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Geometrically, what is the set of *all* linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (a) A point
   (b) A line
   (c) A plane
   (d) All of $\mathbb{R}^3$

9. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following sets has the same span as the set of all three vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? *Mark all that apply.*

   (a) $\{\mathbf{v}_1, \mathbf{v}_2\}$
   (b) $\{\mathbf{v}_2, \mathbf{v}_3\}$

(c) $\{\mathbf{v}_1, \mathbf{v}_3\}$

(d) None of the above

10. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$. Geometrically, what is the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

    (a) A point

    (b) A line

    (c) A plane

    (d) All of $\mathbb{R}^3$

11. **Theorem:** Let A be an $m \times n$ matrix. Then the following statements are logically equivalent. That is, for a particular A, either they are all true statements or they are all false statements.

    (a) For each $\mathbf{b}$ in $\mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.

    (b) Each $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A.

    (c) The columns of A span $\mathbb{R}^m$.

    (d) A has a pivot position on every row.

    If the four statements in this theorem are all true, what does that imply about the relative size of m and n?

    (a) $m \geq n$

    (b) $m \leq n$

    (c) $m = n$

    (d) Nothing. m and n can be any positive integers.

12. Let $A = \begin{bmatrix} 4 & 7 & 2 \\ -1 & 0 & 8 \\ 3 & 5 & 1 \end{bmatrix}$. The entry $a_{2,3}$ is which of the following?

    (a) 0

    (b) 1

    (c) 8

    (d) 5

13. Let $A = \begin{bmatrix} 4 & 7 & 2 \\ -1 & 0 & 8 \\ 3 & 5 & 1 \end{bmatrix}$. Which of the following is equal to $\mathbf{a}_2$?

    (a) $\begin{bmatrix} -1 & 0 & 8 \end{bmatrix}$

    (b) $\begin{bmatrix} 7 \\ 0 \\ 5 \end{bmatrix}$

    (c) 0

    (d) -1

14. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. Which of the following is equal to $5A$?

    (a) $\begin{bmatrix} 10 & 3 \\ 1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & 15 \\ 1 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 15 \\ 5 & 20 \end{bmatrix}$

(d) $\begin{bmatrix} 10 & 3 \\ 1 & 20 \end{bmatrix}$

15. Let $B = \begin{bmatrix} 4 & 7 \\ -1 & 0 \\ 5 & 3 \end{bmatrix}$. Which of the following is equal to B^T?

(a) $\begin{bmatrix} 4 & -1 & 5 \\ 7 & 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 4 \\ 0 & -1 \\ 3 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 3 \\ -1 & 0 \\ 4 & 7 \end{bmatrix}$

(d) B^T cannot be computed.

16. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 7 \\ 0 & 2 \end{bmatrix}$. Which of the following is equal to AB?

(a) $\begin{bmatrix} -2 & 21 \\ 0 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 20 \\ -1 & 15 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 25 \\ 2 & 8 \end{bmatrix}$

(d) AB cannot be computed.

17. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 7 \\ 2 & 6 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$, then which of the following can be computed? (Mark all that apply.)

(a) AB

(b) BA

(c) BC

(d) CB

(e) AC

18. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then $A\mathbf{b}_1$ is described by which of the following?

(a) The first row of AB

(b) The first column of AB

(c) The $1,1$-entry of AB

(d) The first row of BA

19. Suppose you have a simple board game called Quad that consists of four spaces, A, B, C, and D. Movement is as follows:

(a) If you're on space A, you have a 50% chance of moving to space B and a 50% chance of staying on space A.
(b) If you're on space B, you have a 50% chance of moving to space A and a 50% chance of staying on space B.
(c) If you're on space C, you have a 30% chance of moving to space D and a 70% chance of staying on space C.
(d) If you're on space D, you have a 30% chance of moving to space C and a 70% chance of staying on space D.

Is the transition matrix P for this system regular? That is, is there some positive integer power k such that P^k has all positive entries?

(a) Yes
(b) No

20. True or False: If A and B are square matrices with the same dimensions, then $(A+B)(B+A) = A^2 + 2AB + B^2$.

(a) True
(b) False

21. True or False: If $AB = AC$, then $B = C$.

(a) True
(b) False

22. True or False: If $AD = 0$, then either $A = 0$ or $D = 0$.

(a) True
(b) False

23. If A and B are invertible matrices, then what is $(AB)^{-1}$?

(a) $A^{-1}B^{-1}$
(b) $B^{-1}A^{-1}$
(c) AB^{-1}
(d) BA^{-1}
(e) More than one of the above is true.
(f) Just because A and B have inverses, this doesn't mean that AB has an inverse.

24. Which of the following matrices does not have an inverse?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
(b) $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$
(e) All have inverses.

25. Suppose that for a square coefficient matrix A, the matrix equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$. This means that

(a) Matrix A has no inverse.

(b) Matrix A has an inverse.

(c) This tell us nothing about whether A has an inverse.

26. Which of the following is a set of linearly independent vectors?

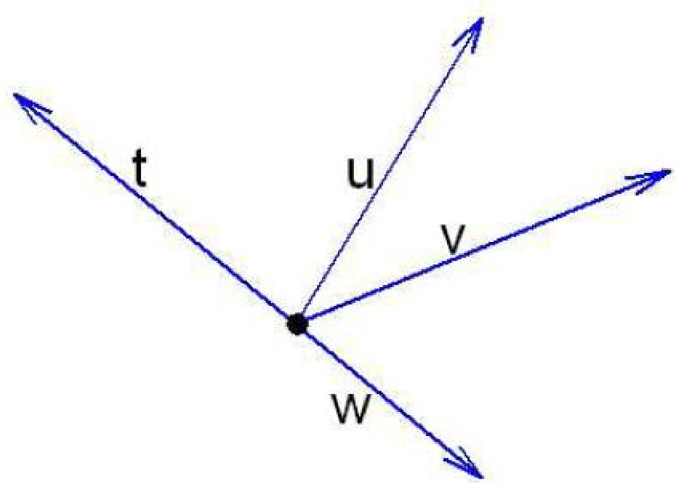
(a) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 6 \\ -12 \\ -21 \end{bmatrix}$

27. Which subsets of the set of vectors shown are linearly dependent? (Mark all that apply.)



(a) $\mathbf{u}, \mathbf{w}$

(b) $\mathbf{t}, \mathbf{w}$

(c) $\mathbf{t}, \mathbf{v}$

(d) $\mathbf{t}, \mathbf{u}, \mathbf{v}$

28. Suppose you wish to determine whether a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. You form the matrix $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix}$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write $\mathbf{v}_2$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_3$, and $\mathbf{v}_4$. Which is a correct linear combination?

(a) $\mathbf{v}_2 = 3\mathbf{v}_3 + \mathbf{v}_4$

(b) $\mathbf{v}_2 = -3\mathbf{v}_3 - \mathbf{v}_4$

(c) $\mathbf{v}_2 = \mathbf{v}_4 - 3\mathbf{v}_3$

(d) $\mathbf{v}_2 = -\mathbf{v}_1 + \mathbf{v}_4$

(e) I don't know.

Answers:

1. (a), (b), and (c)
2. (a), (c), and (d)
3. False
4. True
5. (a), (b), and (c)
6. (c)
7. (a)
8. (c)
9. (a) and (c)
10. (c)
11. (b)
12. (c)
13. (b)
14. (c)
15. (a)
16. (b)
17. (a) and (c)
18. (b)
19. No
20. False
21. False
22. False
23. (b)
24. (b)
25. (b)
26. (c)
27. (b) and (d)
28. (d)