Math 194
Clicker Questions

Unit 1: Systems of Linear Equations

1. Suppose you have a system of linear equations consisting of three equations in two unknowns. How many solutions are possible for such a system? *Mark all that apply.*

   (a) No solutions.

   (b) Exactly one solution.

   (c) Infinitely many solutions.

2. Consider the following system of linear equations.

$$\begin{array}{rcrcrcr} x & + & 3y & + & z & = & 3 \\ 2x & + & 2y & + & 4z & = & -4 \\ 3x & + & y & - & z & = & 9 \end{array}$$

   The corresponding augmented matrix is

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 2 & 4 & -4 \\ 3 & 1 & -1 & 9 \end{bmatrix}.$$

   Which of the following augmented matrices correspond to a system of linear equations with the same solution set? *Mark all that apply.*

   (a) Multiply the first row by 2:

$$\begin{bmatrix} 2 & 6 & 2 & 6 \\ 2 & 2 & 4 & -4 \\ 3 & 1 & -1 & 9 \end{bmatrix}$$

   (b) Multiply the first column by 2:

$$\begin{bmatrix} 2 & 3 & 1 & 3 \\ 4 & 2 & 4 & -4 \\ 6 & 1 & -1 & 9 \end{bmatrix}$$

   (c) Add the first row to the third row:

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 2 & 4 & -4 \\ 4 & 4 & 0 & 12 \end{bmatrix}$$

   (d) Interchange the first and second rows:

$$\begin{bmatrix} 2 & 2 & 4 & -4 \\ 1 & 3 & 1 & 3 \\ 3 & 1 & -1 & 9 \end{bmatrix}$$

   (e) Interchange the first and second columns:

$$\begin{bmatrix} 3 & 1 & 1 & 3 \\ 2 & 2 & 4 & -4 \\ 1 & 3 & -1 & 9 \end{bmatrix}$$

3. Suppose the augmented matrix for a system of three linear equations in four unknowns has three pivot columns. True or False: The system must be consistent.

4. Suppose the augmented matrix for a system of three linear equations in three unknowns has a pivot in each of the first three columns. True or False: The system has exactly one solution.

5. Suppose you row-reduce the augmented matrix for a system of four linear equations in three unknowns and end up with a row of all zeros. How many solutions does the system have? *Mark all that are possible.*

   (a) No solutions
   (b) Exactly one solution
   (c) Infinitely many solutions

6. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Geometrically, what is the set of *all* linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2\}$?

   (a) A point
   (b) A line
   (c) All of $\mathcal{R}^2$

7. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following is *not* a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$
   (d) All of these are linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

8. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Geometrically, what is the set of *all* linear combinations of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (a) A point
   (b) A line
   (c) A plane
   (d) All of $\mathbb{R}^3$

9. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following sets has the same span as the set of all three vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? *Mark all that apply.*

   (a) $\{\mathbf{v}_1, \mathbf{v}_2\}$
   (b) $\{\mathbf{v}_2, \mathbf{v}_3\}$

(c) $\{\mathbf{v}_1, \mathbf{v}_3\}$

(d) None of the above

10. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$. Geometrically, what is the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

    (a) A point

    (b) A line

    (c) A plane

    (d) All of $\mathbb{R}^3$

Answers:

1. (a), (b), and (c)
2. (a), (c), and (d)
3. False
4. True
5. (a), (b), and (c)
6. (c)
7. (a)
8. (c)
9. (a) and (c)
10. (c)