Math 194
The Invertible Matrix Theorem (Unit 2 Version)

Let A be a square $n \times n$ matrix. Then the following statements are equivalent. That is, for a given A, the statements are either all true or all false.

1. A is an invertible matrix.
2. A is row equivalent to the $n \times n$ identity matrix.
3. A has n pivot positions.
4. A has a pivot in each column.
5. A has a pivot in each row.
6. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
7. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
8. The equation $A\mathbf{x} = \mathbf{b}$ has exactly one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
9. The columns of A span $\mathbb{R}^n$.
10. The columns of A are linearly independent.