Math 194
Problem Set 1

1. Solve each of the following systems of linear equations by hand using Gaussian elimination.

   (a)
$$\begin{array}{rcrcrcr} x & - & y & + & 2z & = & 5 \\ 2x & - & 2y & + & 4z & = & 10 \\ 3x & - & 3y & + & 6z & = & 15 \end{array}$$

   (b)
$$\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 0 \\ & & 2y & - & 8z & = & 8 \\ -4x & + & 5y & + & 9z & = & -9 \end{array}$$

2. Given a system of the form
$$\begin{array}{rcrcr} -m_1x & + & y & = & b_1 \\ -m_2x & + & y & = & b_2 \end{array}$$
   where m_1, m_2, b_1, and b_2 are constants,

   (a) Show algebraically that the system will have a unique solution if $m_1 \neq m_2$.

   (b) Show that if $m_1 = m_2$, then the system will have a solution only if $b_1 = b_2$.

   (c) Give a geometric interpretation of parts (a) and (b).

3. Construct three different augmented matrices for linear systems whose solution set is $x_1 = -2$, $x_2 = 1$, and $x_3 = 0$.

4. Suppose you have a simple board game called Triad consisting of three spaces (A, B, and C) arranged in a triangle. Each round you move your playing piece according to the following rule: There's a 50% chance you'll stay where you are, a 30% chance you'll move forward one space, and a 20% chance you'll move backward one space. (Assume that forward = clockwise around the triangle and backward = counterclockwise.)

   (a) Considering this game as a Markov chain, construct the transition matrix P.

   (b) What does the $(2, 3)$ entry in P represent?