Math 115F Math Exam Study Guide

This week's math exam will cover the mathematical concepts and techniques we've explored this semester. The exam will not involve any codebreaking, although some questions on the exam may draw on cryptography for context. You are encouraged to bring a calculator (scientific or graphing) to the test, but you will not be allowed to use a laptop during the test.

You should be able to do each of the tasks listed below and understand the concepts associated with each task.

- Modular Arithmetic
  - Determine if two integers are congruent modulo a given integer *m*.
  - Generate a set of integers all congruent to a given integer *x* modulo a given integer *m*.
  - Simplify or solve a modular arithmetic equation.
  - Calculate *x* MOD *m*, given integers *x* and *m*.
- Prime Numbers
  - Determine if a given number is prime.
  - Find the prime factorization of a given composite number.
  - Determine if two given numbers are relatively prime.
  - Find numbers that are relatively prime to a given number.
- Common Divisors
  - Find the common divisors of a set of integers (as in the Kasiski Test).
  - Determine the greatest common divisor of two integers using the Euclidean Algorithm.
- Combinatorics
  - Calculate the number of permutations of *r* objects from a set of *n* objects.
  - Determine the number of unique permutations of a sequence of letters, with or without repeated letters.
  - Calculate the number of combinations of *r* objects from a set of *n* objects.
  - Calculate the number of possibilities for a given scenario using a mix of permutations and/or combinations.
- Probability
  - Compute probabilities for experiments with equally likely outcomes.
  - Compute probabilities using the basic rules of probability. (See next page.)
- Binary Numbers
  - Convert a number from decimal to binary representation.
  - Convert a number from binary to decimal representation.
  - Add or subtract binary numbers.
- Logical Reasoning
  - Prove a simple result in one of the above areas using one or more given theorems.

Basic Rules of Probability

- SUM RULE: If events *A* and *B* are mutually exclusive, then the probability of *A* **or** *B* occurring equals P(*A*) + P(*B*).
- PRODUCT RULE: If events *A* and *B* are independent, then the probability of *A* **and** *B* occurring is P(*A*)·P(*B*).
- COMPLEMENT RULE: The probability of event *A* **not** occurring is 1-P(*A*).

Practice Problems

1. Determine five solutions to each of the following equations.
    a. $x - 4 \equiv 5 \pmod{26}$
    b. $x + 23 \equiv 1 \pmod{4}$
    c. $5x \equiv 1 \pmod{8}$
    d. $3x + 1 \equiv 4 \pmod{5}$

2. Calculate the following.
    a. 14 MOD 3
    b. 130 MOD 26
    c. -1 MOD 5
    d. -258 MOD 16

3. Determine the prime factorization of the following numbers.
    a. 961
    b. 2310
    c. 6517

4. Which of the following pairs of numbers are relatively prime?
    a. 45 and 54
    b. 64 and 81
    c. 105 and 122
    d. 1155 and 1729

5. Find three numbers that are relatively prime to each of the following numbers.
    a. 75
    b. 120
    c. 310
    d. 512

6. Find all common divisors for each of the following sets of numbers.
    a. 42, 70, 126, and 154
    b. 50, 125, 275, and 300

c. 52, 130, 182, and 468

7. Use the Euclidean algorithm to find the greatest common divisor of each pair of integers. That is, find gcd(a, b).
    a. a = 667 and b = 437
    b. a = 3001 and b = 541
    c. a = 77897 and b = 3721

8. For each of the following pairs a and b, find integers s and t such that $as + bt$ = gcd(a, b).
    a. a = 667 and b = 437
    b. a = 3001 and b = 541
    c. a = 77897 and b = 3721

9. If the letters B, C, D, F, G, H, and J are written on seven index cards…
    a. How many three-letter "words" can be formed?
    b. How many five-letter "words"?
    c. In how many ways can three of these cards be selected?
    d. In how many ways can five of them be selected?

10. Given a standard 52-card deck (that is, cards ranked Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King, in each of four different suits—hearts, diamonds, clubs, and spades), determine the number of each type of hand listed below that are possible on a 5-card draw.
    a. Full House – 3 cards of one rank, 2 cards of another rank
    b. Flush – 5 cards of the same suit
    c. Straight – 5 cards of consecutive ranks (ex.: 8, 9, 10, Jack, Queen)
    d. Three-of-a-Kind – 3 cards of one rank, 2 cards of other ranks

11. Tennessee auto license plates have three letters followed by three digits.
    a. How many different Tennessee plates are possible?
    b. If two Tennessee plates are selected at random, what is the probability that they will have the same three digits?
    c. How many different Tennessee plates include the letters Q, X, and Z?
    d. What is the probability that a randomly selected Tennessee plate will include the letter D?

12. On a certain multiple-choice exam, there are five questions, each with four choices. Bubba has not studied for the exam at all and decides to guess answers at random. What is the probability that…
    a. The first question he gets right is the 5^{th} question?
    b. He gets all questions right?
    c. He gets at least one question right?

13. Find the decimal representation of each of the following binary numbers. (Spaces added for readability.)
    a. 100 001
    b. 110 000
    c. 10 101 011
    d. 111 111 111

14. Find the binary representation of each of the following decimal numbers.
    a. 64
    b. 129
    c. 53
    d. 279

15. Using binary representations, calculate $a + b$ and $a - b$.
    a. a = 11 011, b = 10 101
    b. a = 1 111 11, b = 101

16. Decimal representations use base 10. Binary representations use base 2. Find the decimal representation of each of the following numbers represented in base 3.
    a. 201
    b. 111
    c. 21 001