**Problem Set 8**

1. Suppose you're given the following cipher:

   15-17-22-11-1-22-24-31

   You first think that each letter in the plaintext has been replaced by the number that represents its place in the alphabet, so that 15 = O, 17 = Q, 22 = V, 11 = K, 1 = A, and so on. The fact that OQVKA doesn't spell anything worries you, but you're more worried by the fact that the last number in the cipher doesn't work at all. There aren't 31 letters in the alphabet!

   You find a clue, however, that leads you to believe that you should interpret these numbers as given in base eight[1], not in the usual base-ten decimal system. That would mean 15 = 1*8 + 5 = 13, which corresponds to the letter M.

   If indeed these numbers are written in base eight, then find the plaintext for this cipher[2].

2. Argue that there are at least two people in Columbus, Ohio, who have the same three initials. (Use counting techniques we've discussed in class in your argument!)

3. This question gets at the notion of computational complexity.
   a. Consider the problem of squaring each of the first *n* integers and adding the result: $1^2 + 2^2 + 3_3 + \ldots + n^2$. In terms of *n*, how many arithmetic operations (multiplications and additions) are required to perform this computation?
   b. It can be shown that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$. How many arithmetic operations (multiplications and additions) are required to perform this computation?
   c. For what values of *n* is the method in part (b) faster computationally than the method in part (a)?

[1] Base eight works on the principle that the rightmost digit tells you how many ones you have, the next digit over tells you how many 8s you have, and the next digit over, if you have one, tells you how many 64s you have, since $8^2$=64.

[2] This problem was adapted from *The Beekeeper's Apprentice* by Laurie R. King. In this novel, Sherlock Holmes takes on a 15-year-old girl as his apprentice in his retirement years.